Material Flow Analysis
Method and applications

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MSE-433 Towards sustainable materials:
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Goals of this lecture

- Get an understanding of the method of Material Flow Analysis (MFA)
- Get to know some examples in sustainability areas where it has been applied

Material Flow Analysis

- Origins
- Method

Origin of material flow analysis

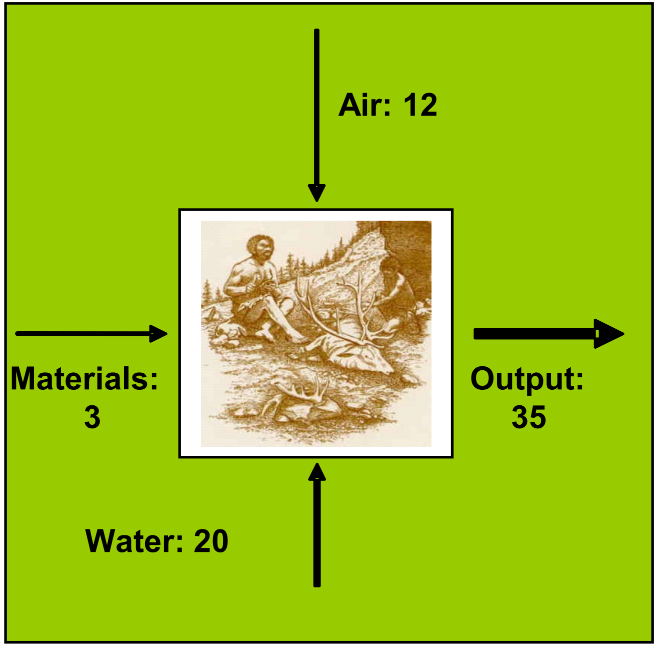
Every industrial chemical process is designed to produce economically a desired product from a variety of starting materials through a succession of treatment steps ..

Figure 1.1 Typical chemical process.

- **A. Lavoisier (1743 – 1794)**
  Law of mass conservation
- **O. Levenspiel (1926 - 2017)**
  Chemical production processes

Changes in the resource use
(kg/cap*year)

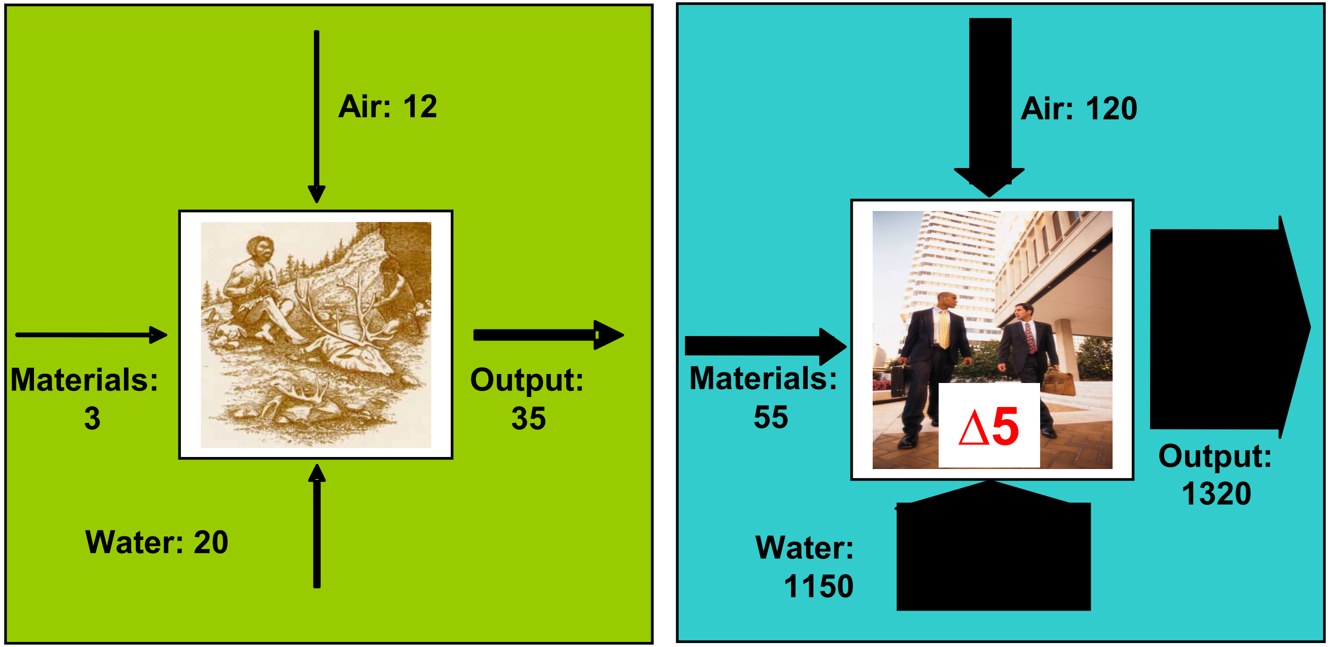
Hunters and gatherers



Changes in the resource use
(kg/cap*year)

Hunters and gatherers

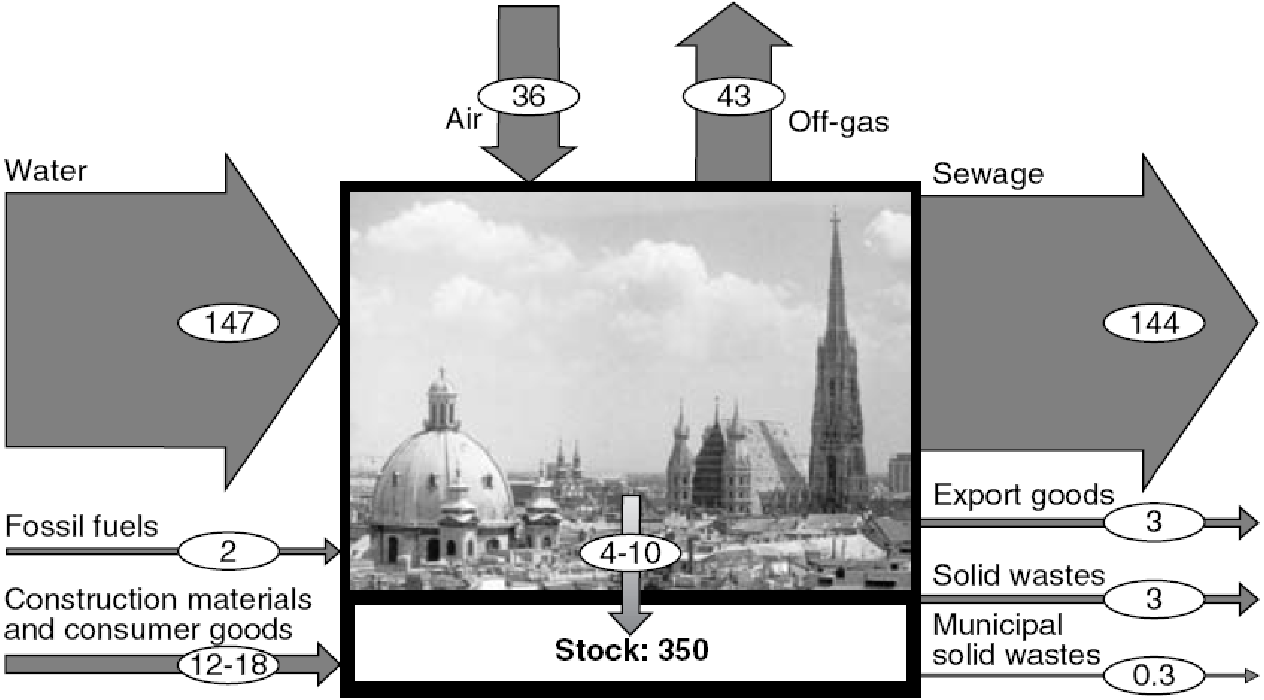
Industrialised societies



Binder, 2006; after Fischer-Kowalski et al., 1997

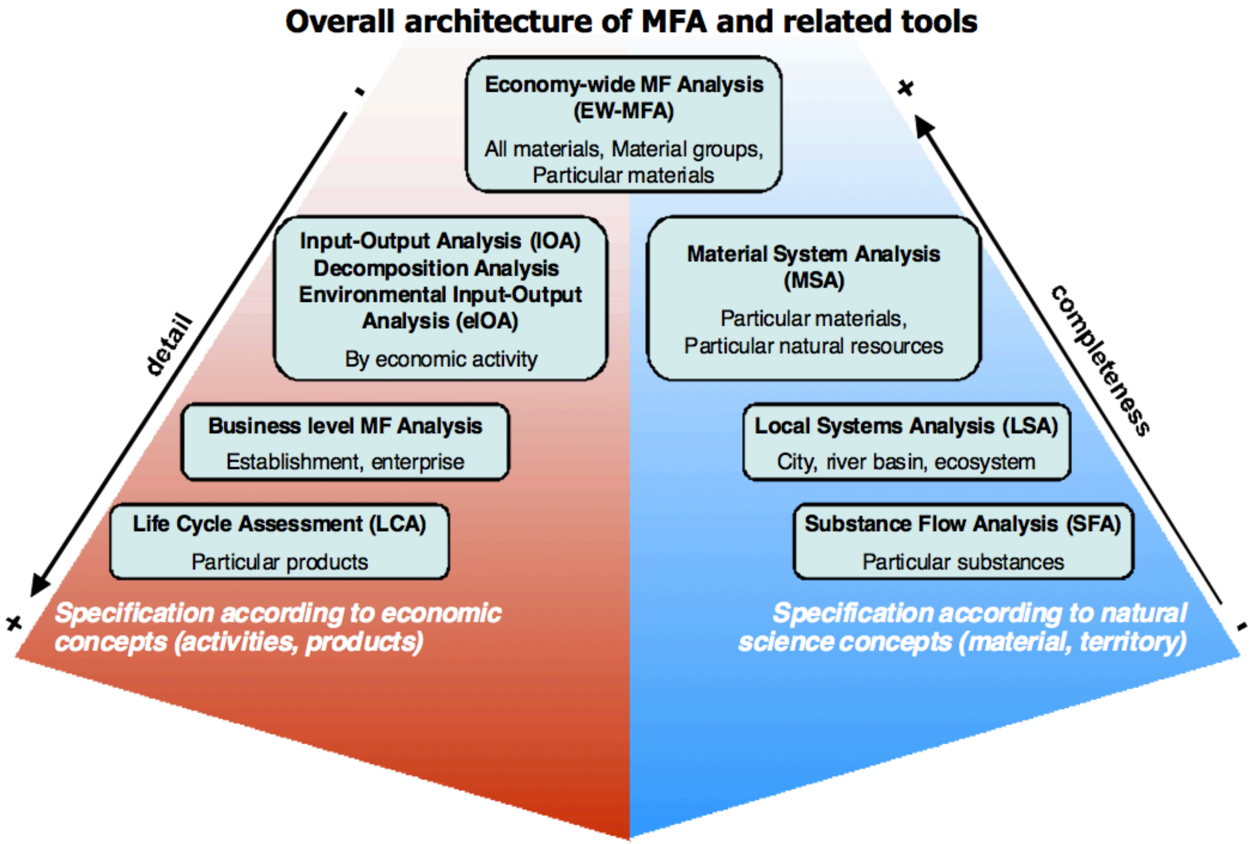
Urban Metabolism of Vienna in 1990is

(tons/cap/year)



Brunner & Rechberger (2004), Kennedy (2007)

Hierarchy of MFA – related tools



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OECD, 2008

Definition: Material flow analysis (MFA)

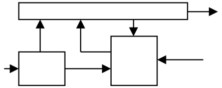
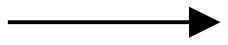
MFA is a systematic assessment of the **flows and stocks of materials / elements** within a **system** defined in **space** and **time**.

It connects **sources**, **pathways**, and **intermediate** and **final sinks**.

- Overview of a system and the relevant system stocks and flows
- Quantifying substance and/or energy flows
    - a specific city / region / nation (space)
    - a specific period of time
- Origin (sources) / transformation processes
- Principle of **mass and energy conservation**

(Source: Brunner and Rechberger, 2004)

Key terms in MFA

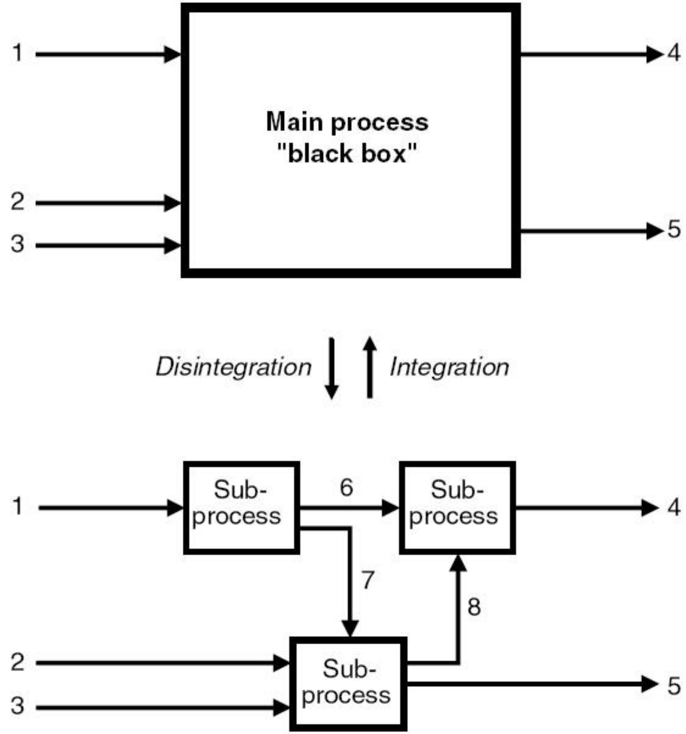
Key terms	Definition	Graphical representation	Mathematical interpretation
MFA system	Open system composed of processes and goods, through which material and energy flow		A specifically defined spatial and temporal unit in which the material and energy flows are measured
System boundary	Delimits the MFA system		Defines the MFA system geographically and within time
Process (stock)	Transport, transformation or depositing of elements and goods		Balance volume (a spatial unit which is balanced for a specific time period and for which mass conservation applies)
Element	Chemical elements or compounds		Elements (i.e., components of materials)

Binder, 2012, after Baccini and Bader 1996

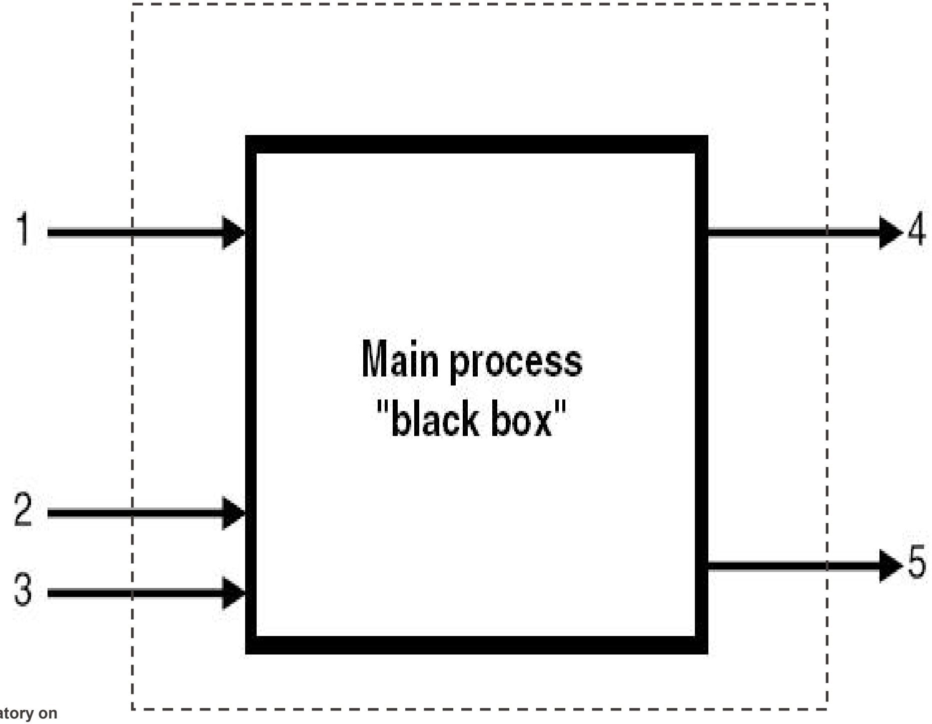
MFA definitions: stocks and flows

- **Stock:** material or substance reservoir inside the system at hand (unit of mass).
- **Flow:** material or substance flowing from one process to another: flows (mass/time) and fluxes (mass/time and section).
  - Flow into the system: input
  - Flow out of the system: output

MFA definitions: processes



MFA definitions: the mass balance

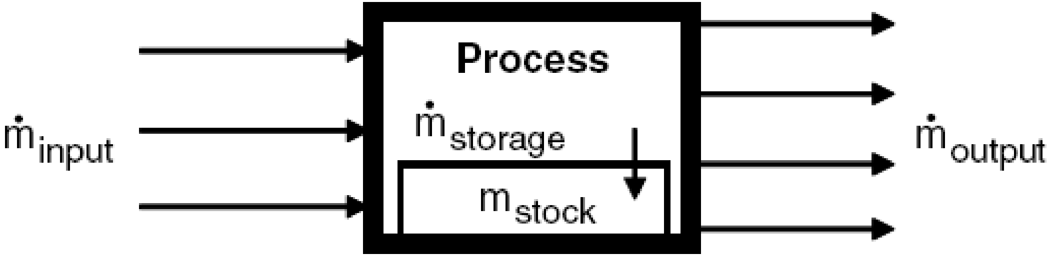


If there is no stock:

Input mass = Output mass

$$m_{i,1} + m_{i,2} + m_{i,3} = m_{o,4} + m_{o,5}$$

MFA definitions: stocks



$$\sum_{k_I} \dot{m}_{input} = \sum_{k_O} \dot{m}_{output} + \dot{m}_{storage}$$

Brunner & Rechberger 2004

MFA definitions: Geogenic vs anthropogenic stocks

Examples of geogenic and anthropogenic stocks ?

MFA definitions: Geogenic vs anthropogenic stocks

Geogenic stocks

Surface and ground water
Quarries
Metal ores
Coal seams
Oil & gas fields
Etc.

Anthropogenic stocks

Concrete and steel in buildings
Copper wires in buildings and durable goods
Urban mines (e.g. landfills)
Etc.

[Source: Ökologische Systemanalyse- Teil 1: SFA, Stefan Rubli]

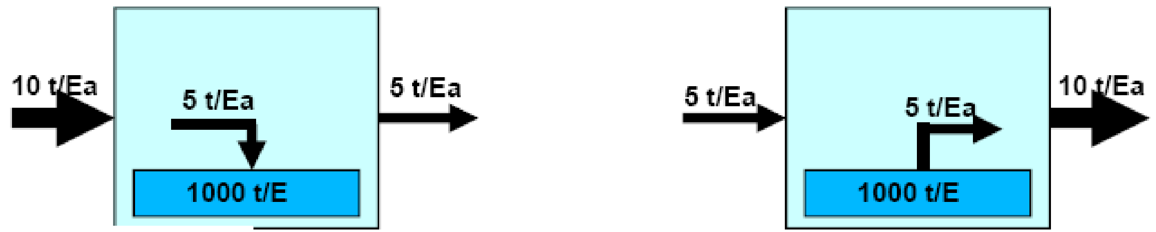
MFA definitions: stocks

Stock: A quantity of material in a process

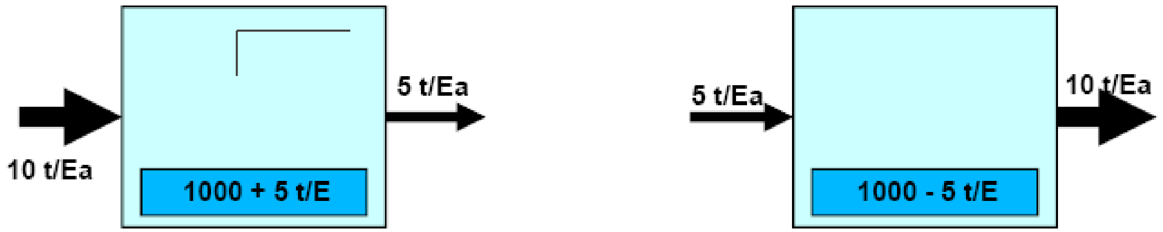
Variation of stock:

Difference between the Inputs and the Outputs of a process between two time steps

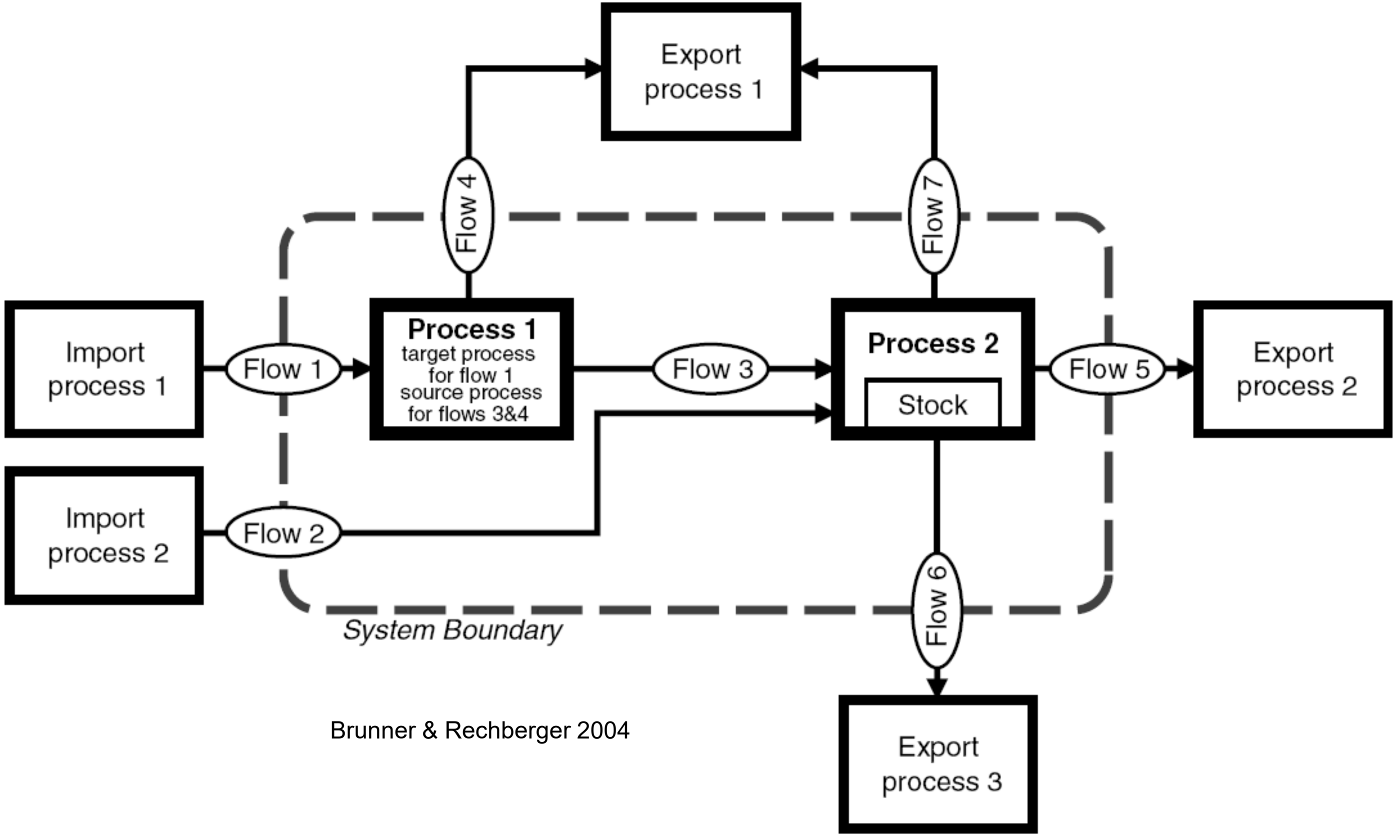
Increase in stock **Decrease in stock**



Other representation



MFA definitions: representation



Brunner & Rechberger 2004

MFA definitions: the time parameter

- **Steady state**: constant flows and stocks
  $dM_{stock}/dt = 0$ and $M_{stock}(0) = M_{stock}(t)$

- **Quasi stationary**: constant flows and linear (de)growth of stocks: $dM_{stock}/dt = c$ and $M_{stock}(0) \neq M_{stock}(t)$

- **Dynamic**: System state at t is a function of the state a t-1.

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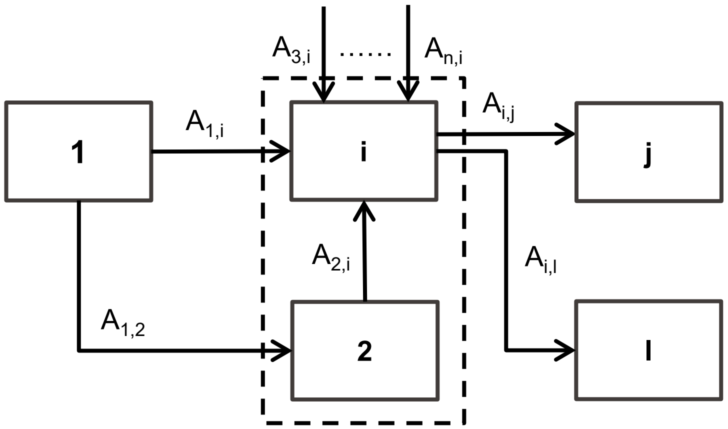
MFA Definitions: Transfer coefficients (I)

- Definition: Transfer coefficient *k*
  - Describes the division of substances/goods leaving any given process
  - Denotes the relative proportion of the total input to process *i* that flows into process *j*.

$$k_{i,j} = \frac{A_{i,j}}{\sum_n A_{n,i}}$$



- In the steady state case:

$$\sum_m k_{i,m} = 1$$

$$k_{i,j} + k_{i,l} = 1$$

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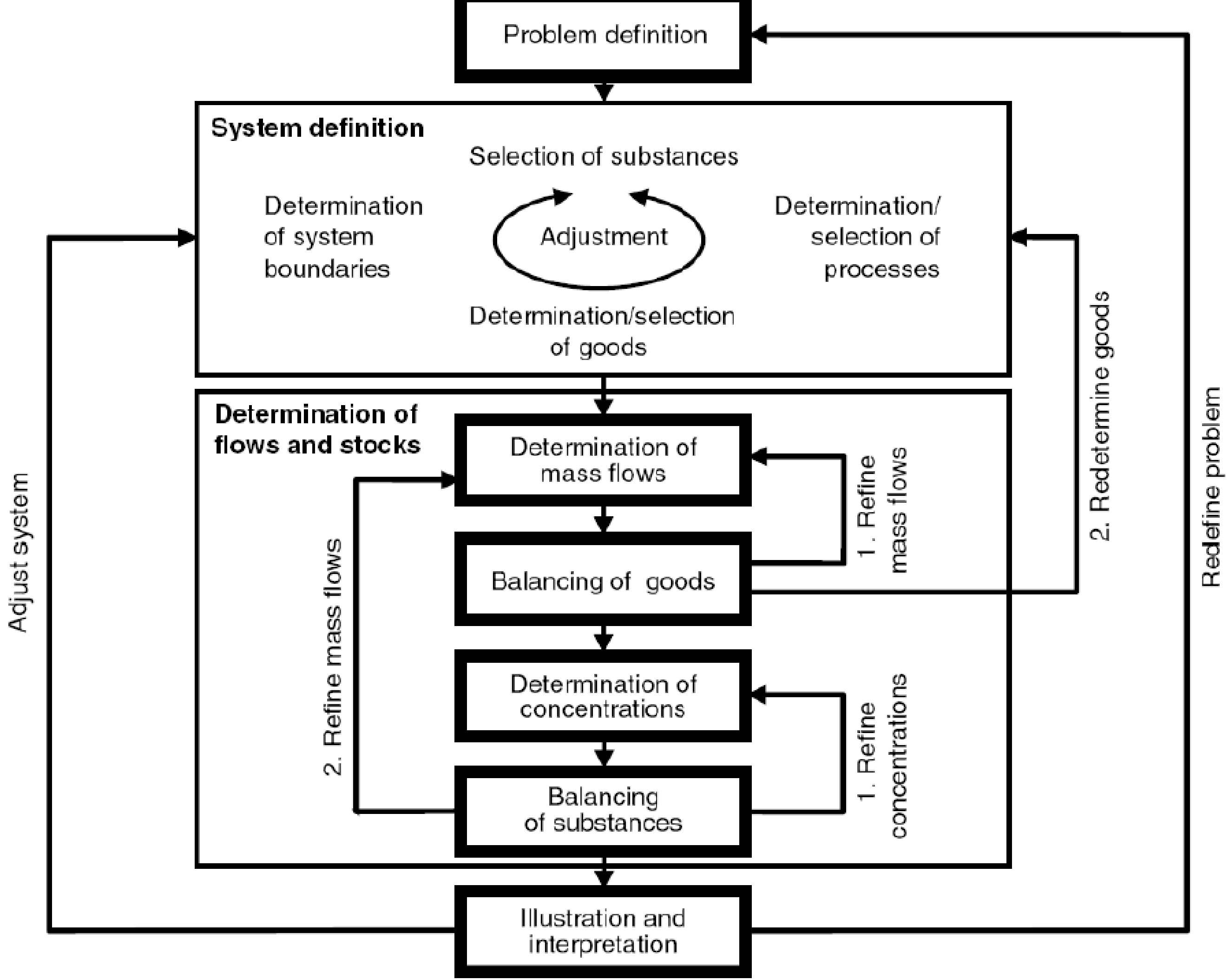
MFA definitions: methodology

1. Define the system unknowns
2. Setup the system of equations: conservation of mass
3. Complete the system of equations with specific relationship, e.g. system parameters
4. Solve the system of equations
5. Analyze the results and check for errors

MFA methodology



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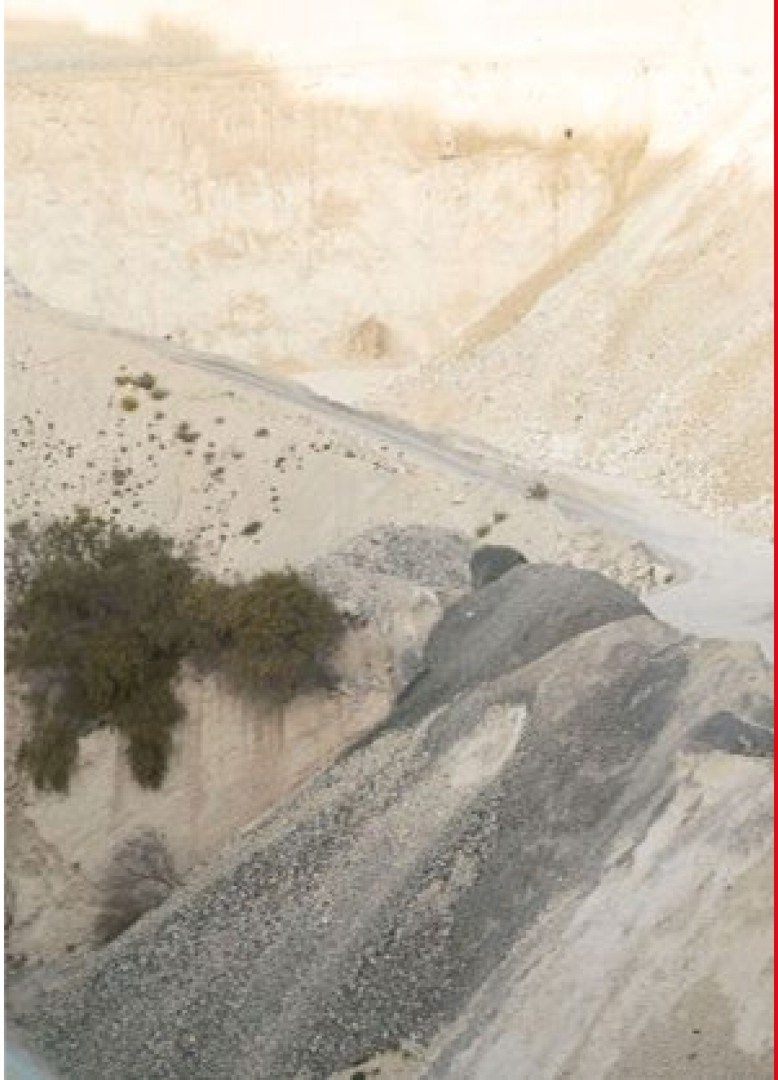

Copper flows at the global level

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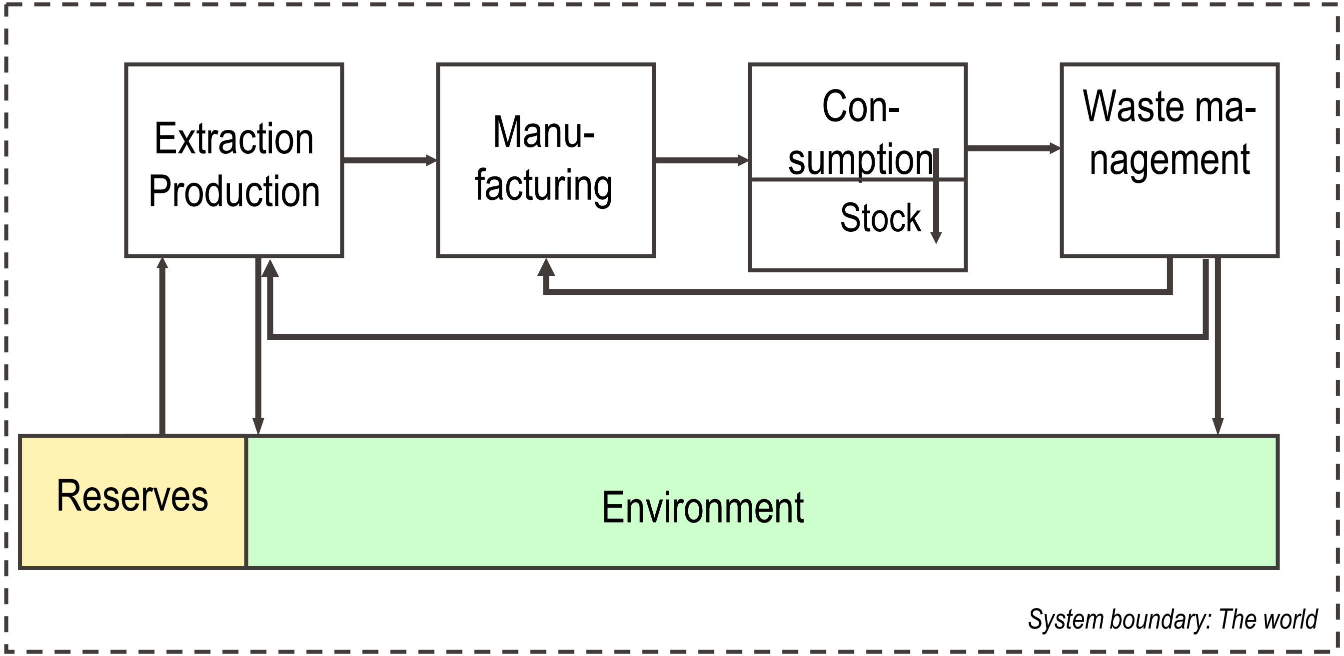
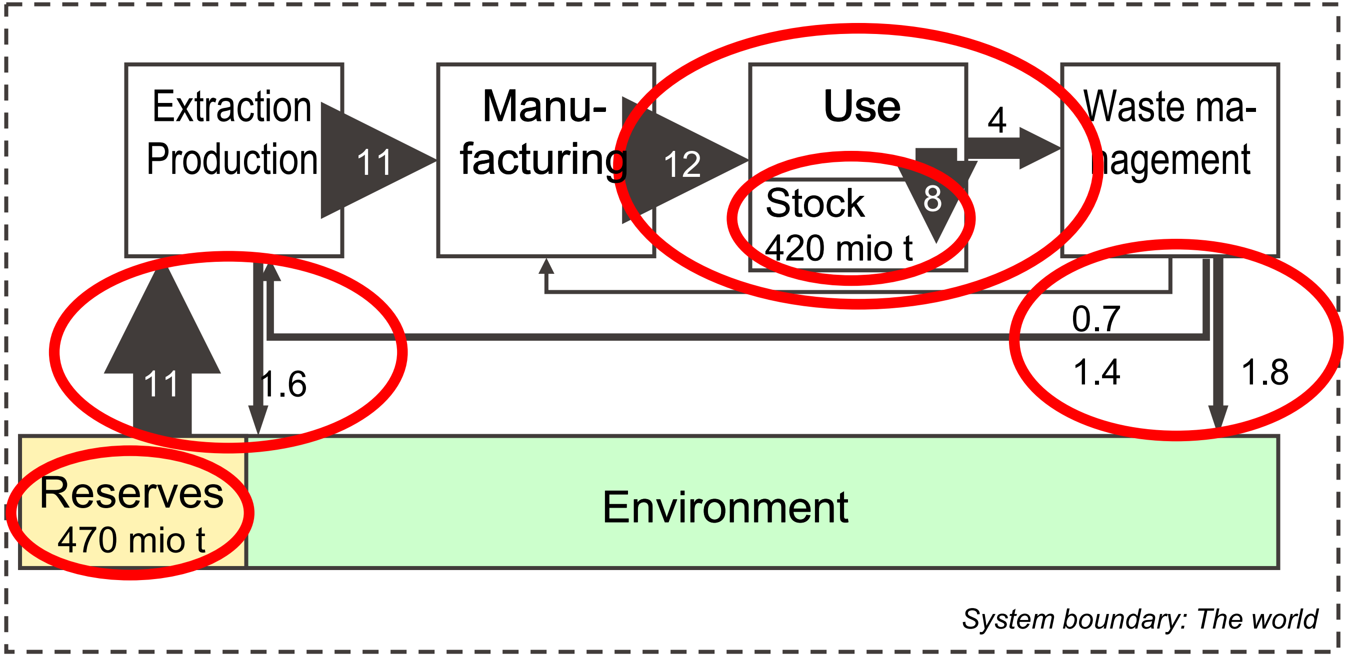
MFA at world level



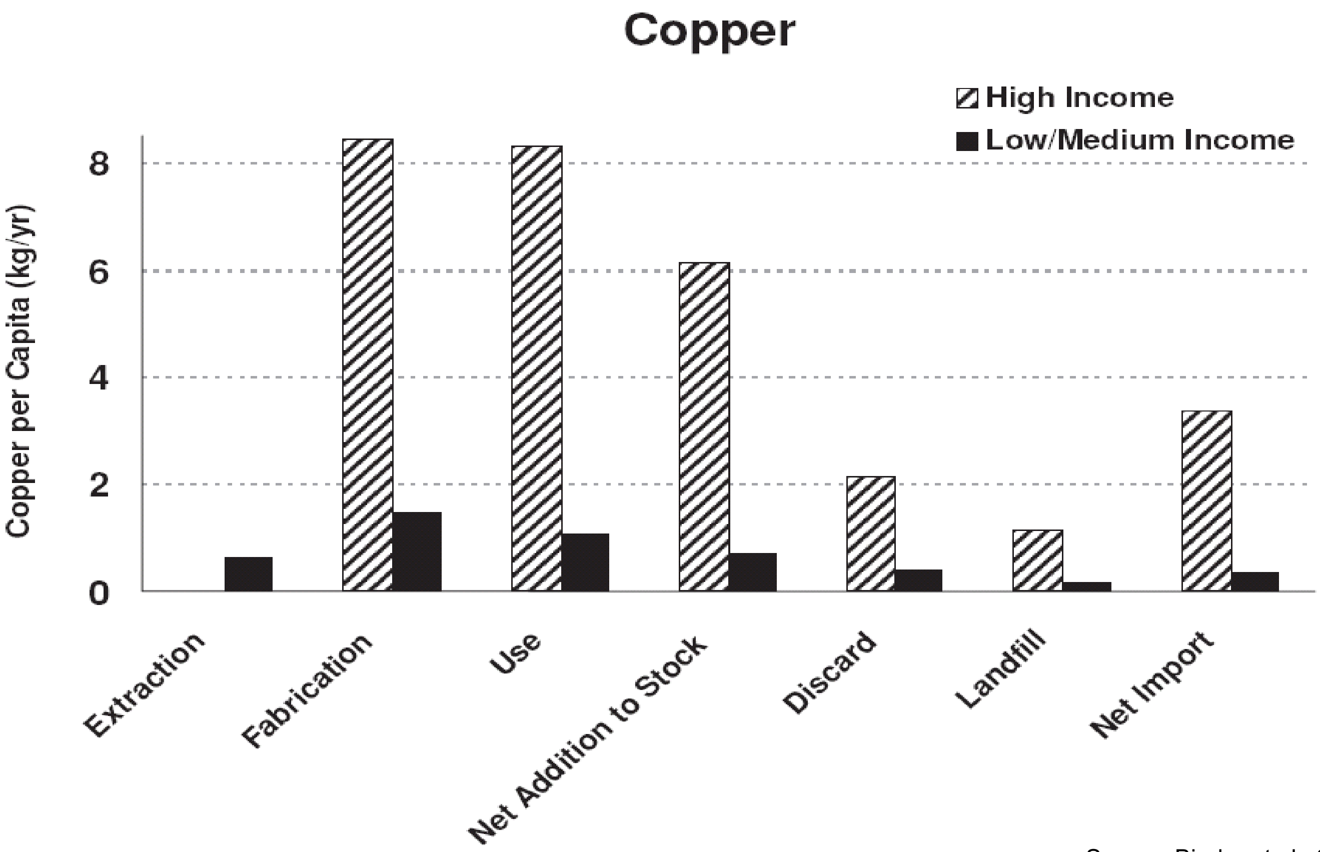
Inspired by Graedel et al., 2004

Global copper flows (Tg/year; rounded values)



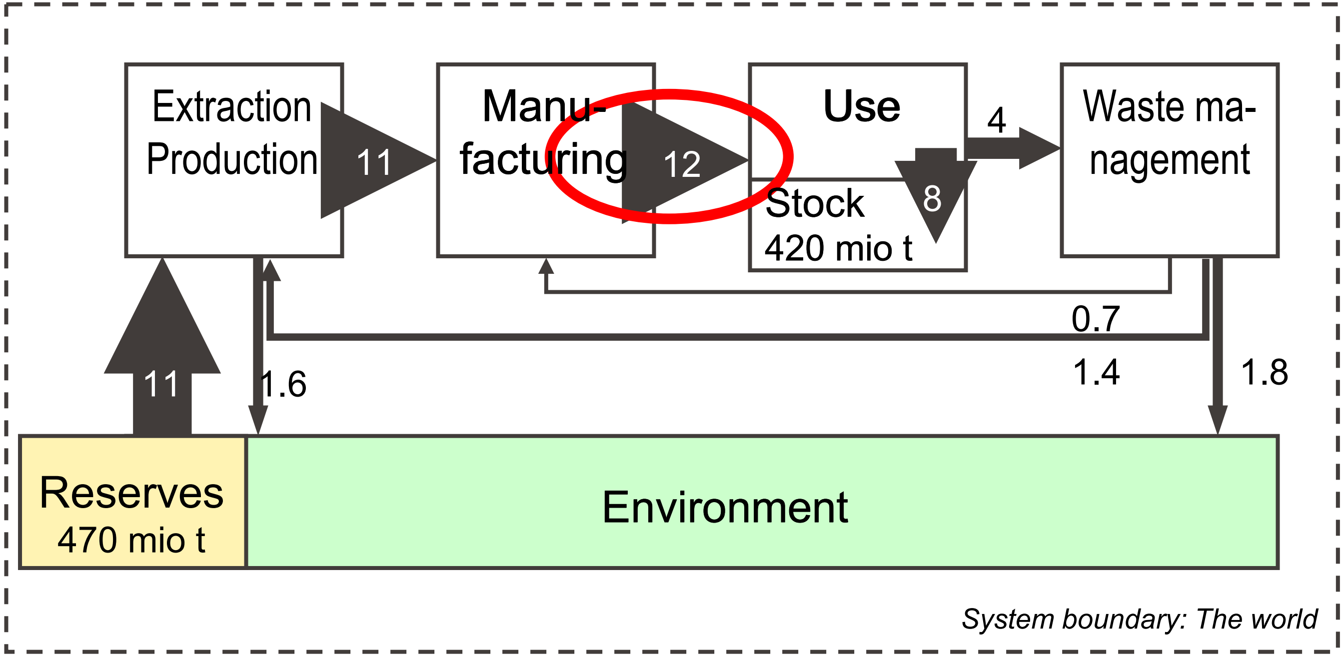
After Graedel et al., 2004

Copper flows North-South



Source: Binder et al., 2006

Global copper flows (Tg/year; rounded values)



After Graedel et al., 2004

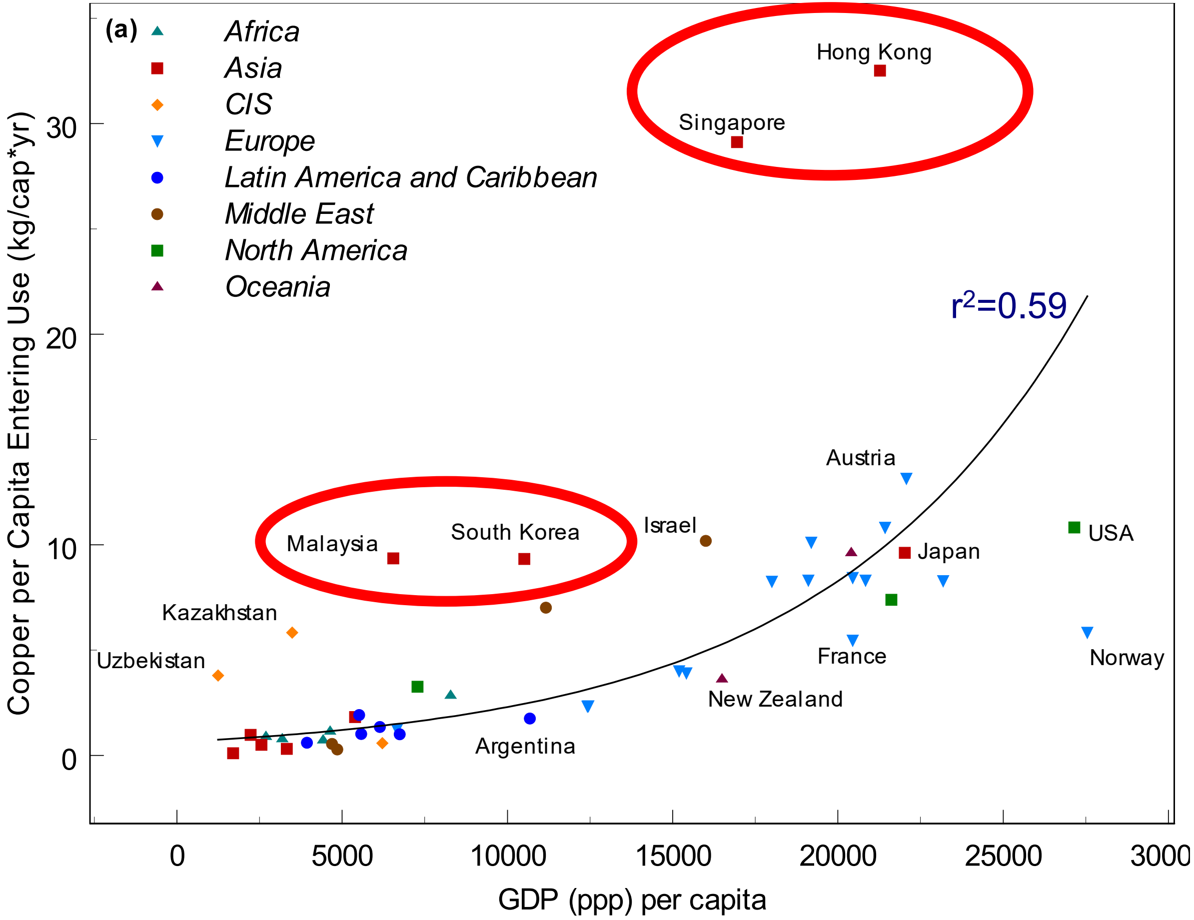
Explanatory variables

Variable group	Explanatory variables
Wealth	GDP per capita (PPP)
Urbanisation & Infrastructure	Urban population (% (%) Telephone lines (per 1,000 people)
Industry	Machinery and transport equipment (% of value added in manufacturing)
Private consumption	Cars*; TV*; Computer* high correlation with GDP

* per 1000 people

Binder, Graedel, Reck, 2006

Copper demand and GDP (1994)



Binder, Graedel, Reck, 2006

Multiple regression for copper demand (1994)

Significant variables

- GDP (PPP)
- % of value added in manufacturing (MAN)
- % urban population (URB)

$$\ln\left[\mathrm{de\,Cu}\right] = -2 + 9 * 10^{-5} * \mathrm{GDP} + 0.04 * \mathrm{MAN} + 0.016 * \mathrm{URB}$$

$r\,2 = 0.788$, adjusted $r\,2 = 0.772$

Binder, Graedel, Reck, 2006

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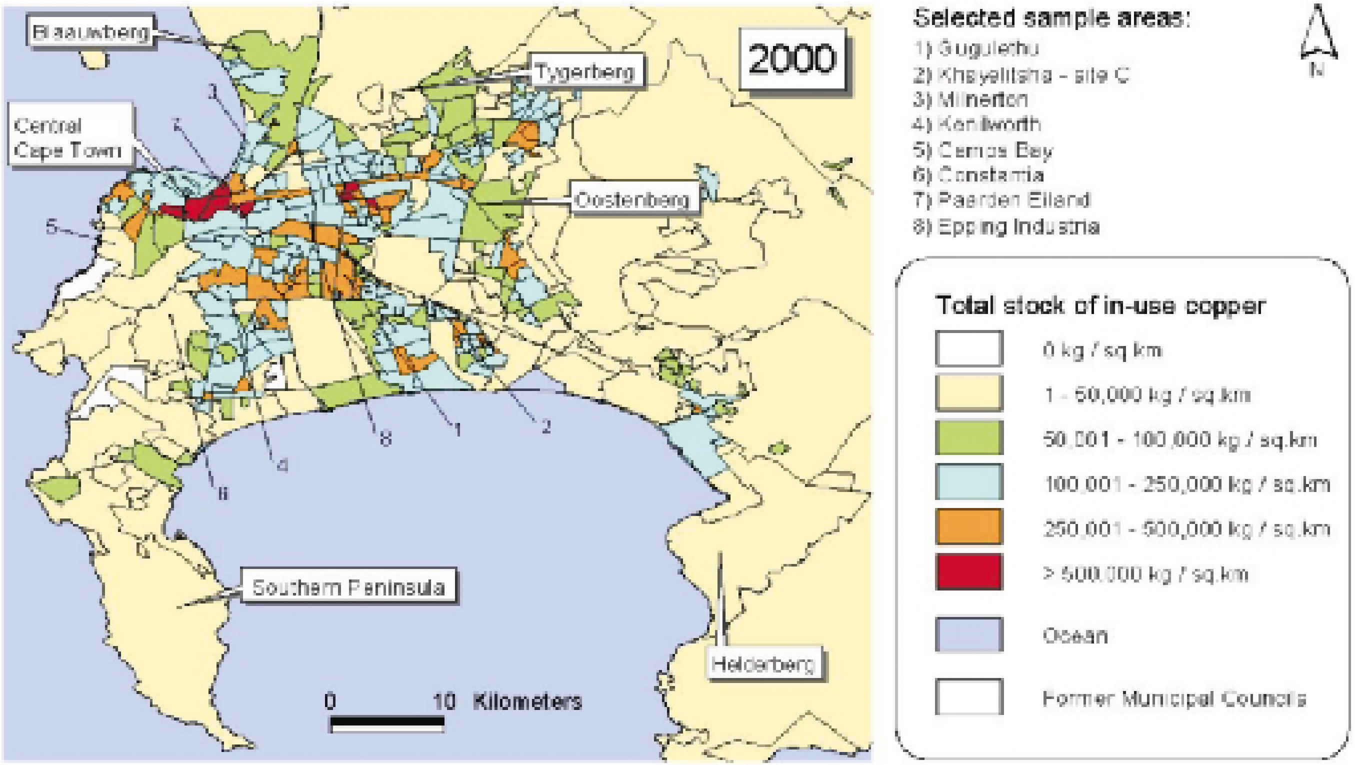
Urban copper reserves in Cape Town (2000)



Source: Graedel at al., 2005

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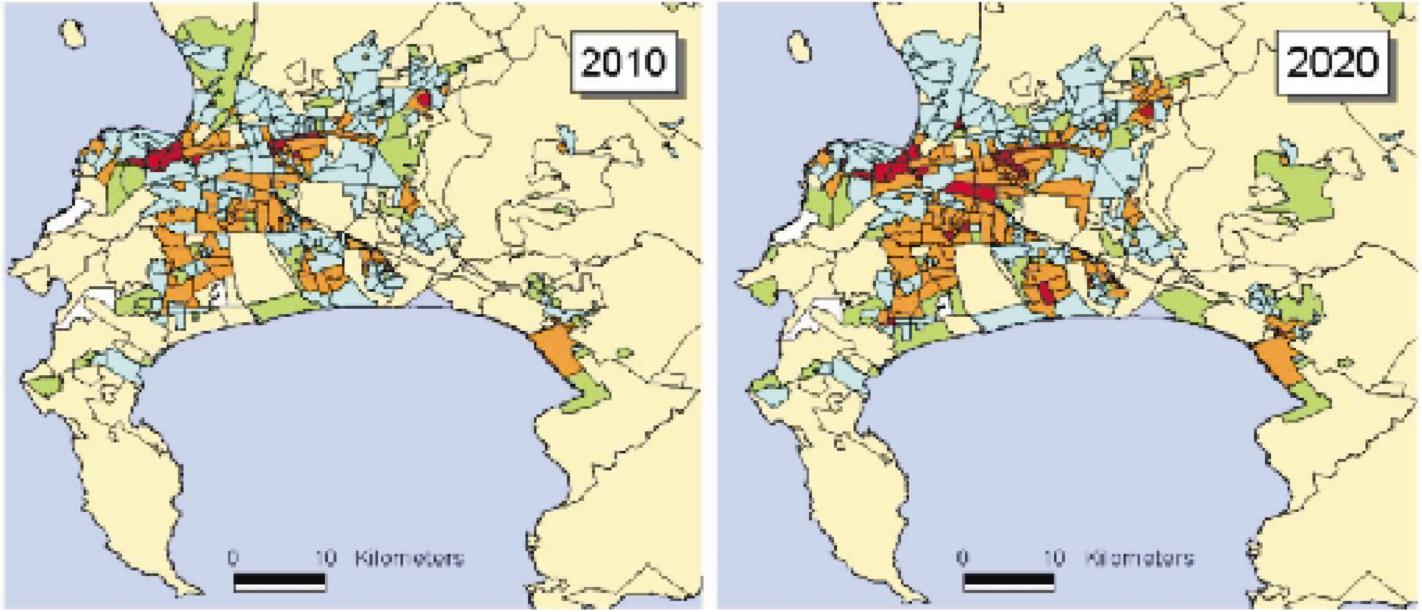
Urban copper reserves and their development



Source: Graedel at al., 2005

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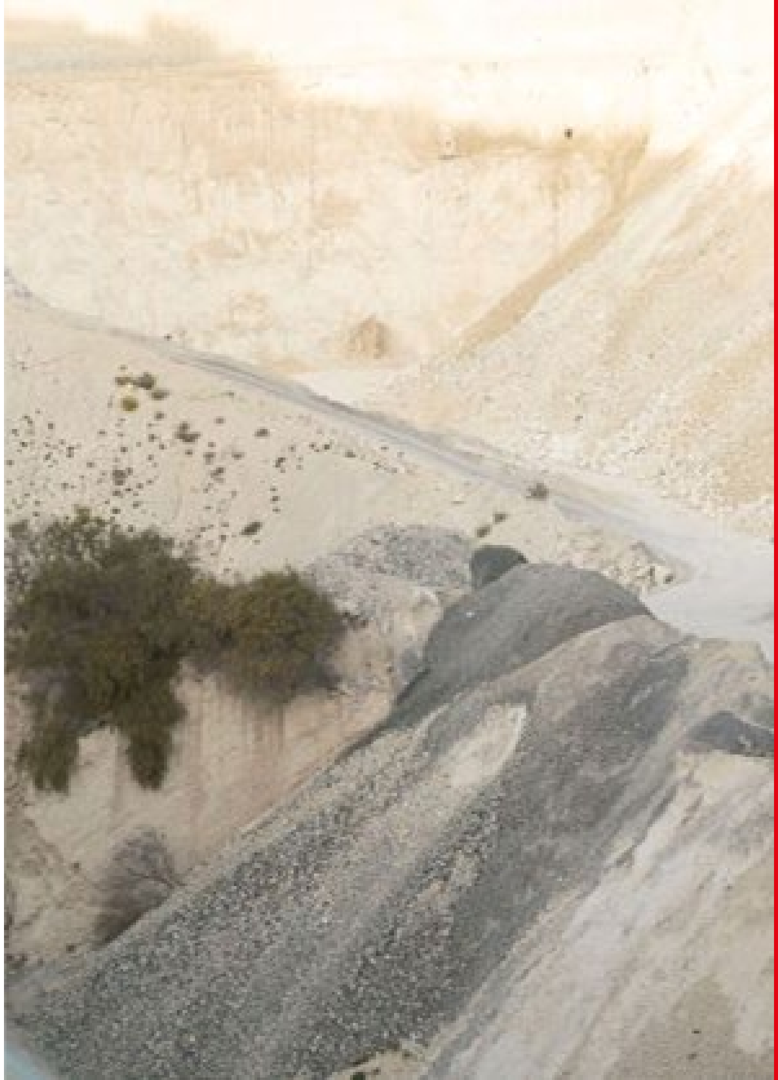
Summary

- Anthropogenic stock of copper is in the same order of magnitude than the geogenic stock – urban stock mining !
- Worldwide copper stock is still growing driven by urbanization and related infrastructure needs, industry and private consumption.
- Losses to the environment are still high, recycling rates, thus, rather low
- MFA allows for quantifying the stock and flows and identifying the areas of intervention



Using MFA to identify innovation potential

Phosphorus management
in Switzerland

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Motivation: Phosphorous

- Essential non-substitutable nutrient for humans and animals
- P-reserves of increasingly bad quality (Uranium, Cadmium)
- Potential geopolitical shortage
- Increasing need for a sustainable P-management

➢ **How can Switzerland manage its P-flows more sustainably?**

Zapata und Roy 2004

Methods: Material flow analysis [1]

- **Spatial boundary:** Switzerland
- **Temporal boundary:** years 2006 / 2011 / 2015
- **System**
  - **6 processes (subsystems):**

    Agriculture animals / husbandry
    Agriculture plants / cultivation
    Chemical industry
    Household and industry
    Waste management
    Water bodies
  - **5 stocks**
  - **84 flows**

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Methods: Material flow analysis [2]

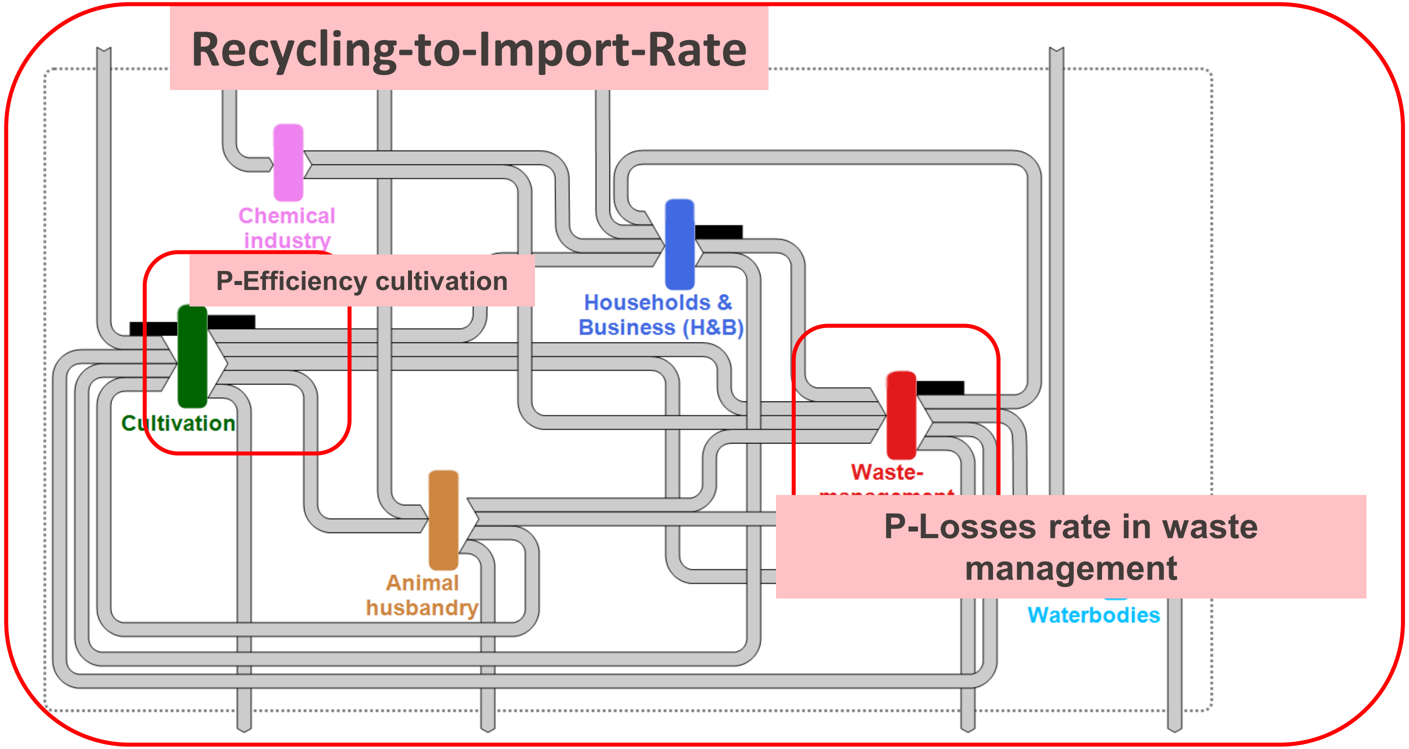
Data sources

- Literature / expert interviews
- Uncertainty ranges

Calculations & software

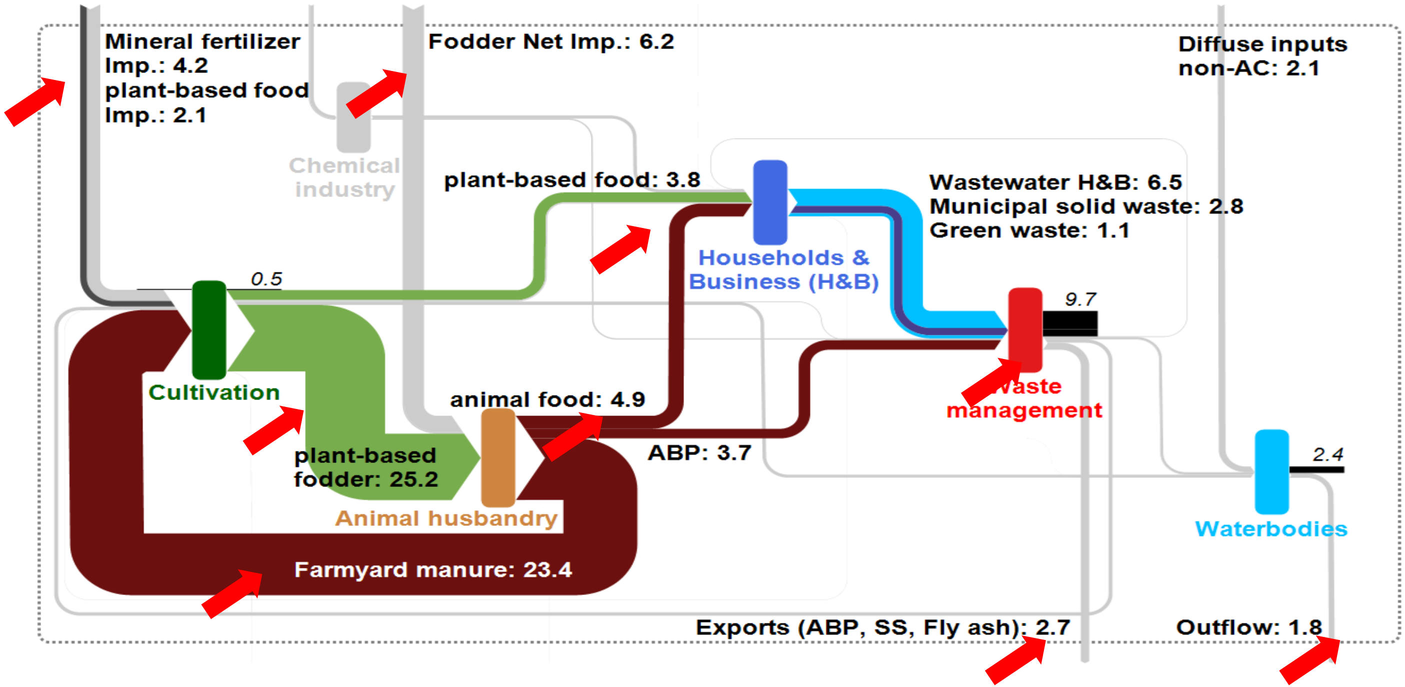
- Error propagation for flows
- STAN

Processes and indicators



Source : Binder & Mehr, 2017

P-flows in CH 2015

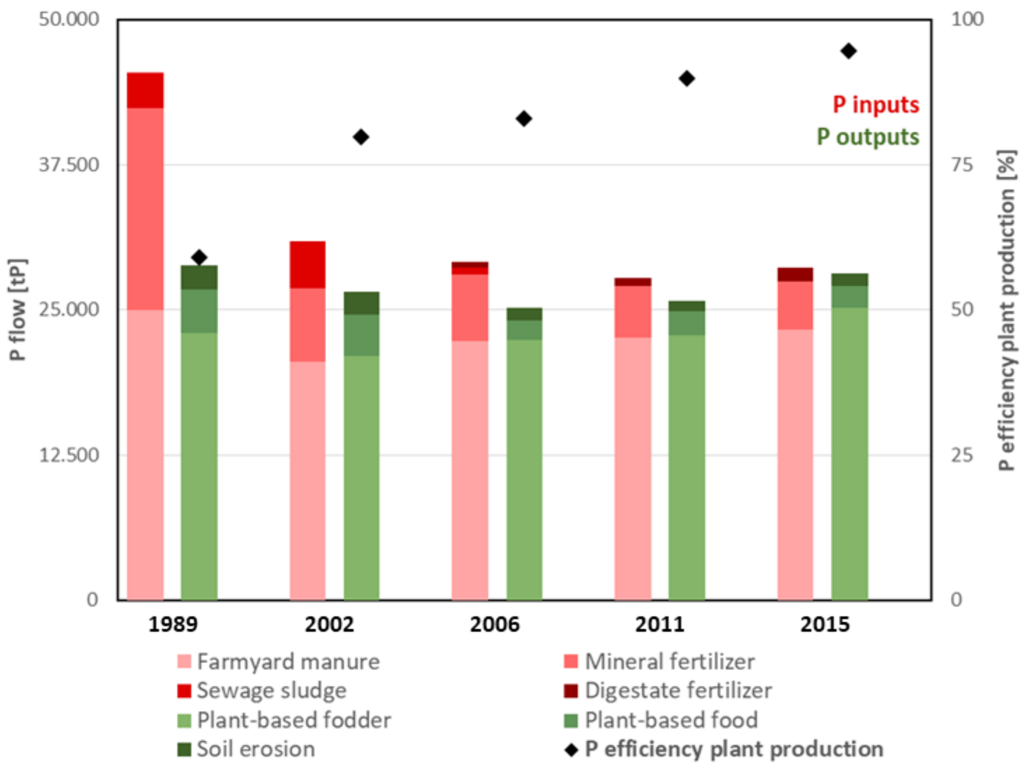


Import ~ 14'000 t P/year (90% agriculture)
Export ~ 4'500 t P/year

→ «**Losses** in deposit» ~10'000 t P/year (93% in landfills and cement industry)

Source : Mehr et al, 2018

P-flows in agriculture



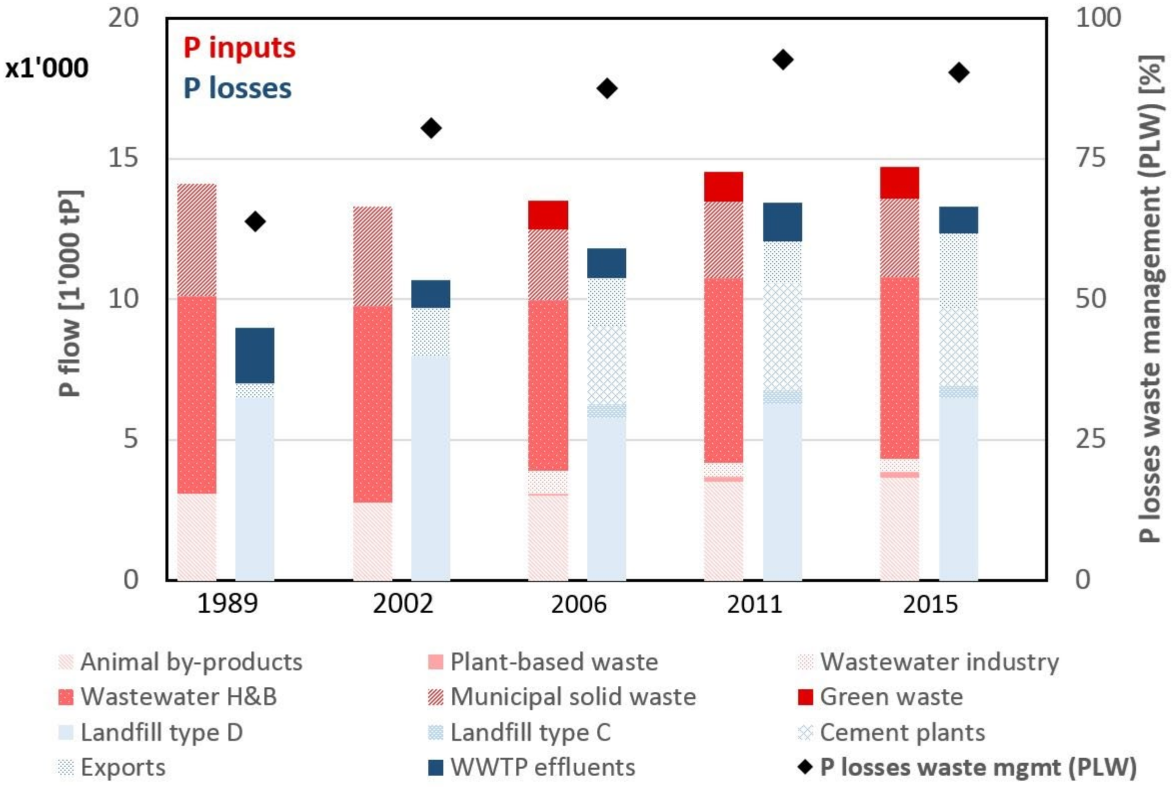
Environmental issues: eutrophication and soil overfertilization

1993: Ordinance of direct payments

1999: Proof of ecological performance (ÖLN)

Based on own research; Binder et al. 2009; Binder & Jedelhauser 2014; Lamprecht et al. 2011

P-flows in waste management



Health issue: BSE
Soil contamination

1990: BSE: Ban meat and bone meal as animal feed

2006: Ban of sewage sludge fertilization

Environmental issue:
P- scarcity and loss of resource

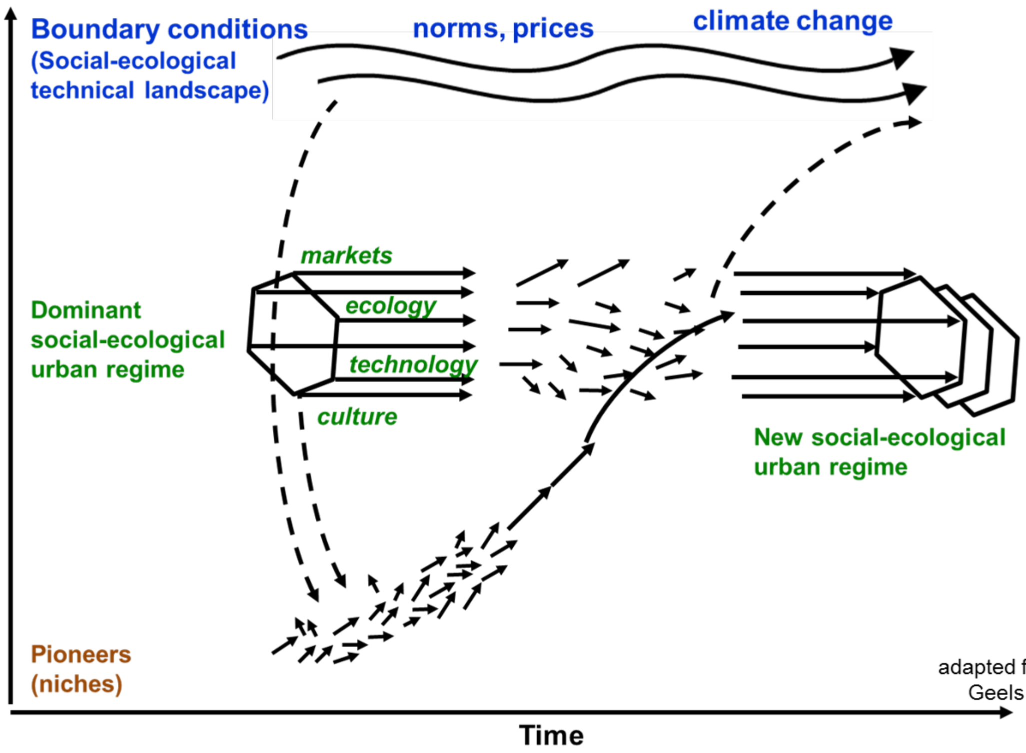
2007: Abrupt rise of P-price

2016: VVEA Ordinance on Avoidance and Disposal of Waste

Source : Mehr et al, 2018

Using MFA to assess ex-ante the impact of innovations and interventions

Transitions occur at different levels



adapted from Geels 2002;
Geels and Schot, 2007

Scenarios

Scenario	Context	Assumptions
Balanced and healthy human diet	Government initiative: Swiss Nutrition Strategy 2017-2024	Food recommendations according to FSVO
Implementation of VVEA	Legislative intervention: VVEA	Full implementation of VVEA, i.e. P recovery from wastewater, sewage sludge or sewage sludge ashes and utilization of P in meat and bone meal
Urine separation	Research & pilot projects	Separate collection and recycling of 20% of total urine on the household & businesses level

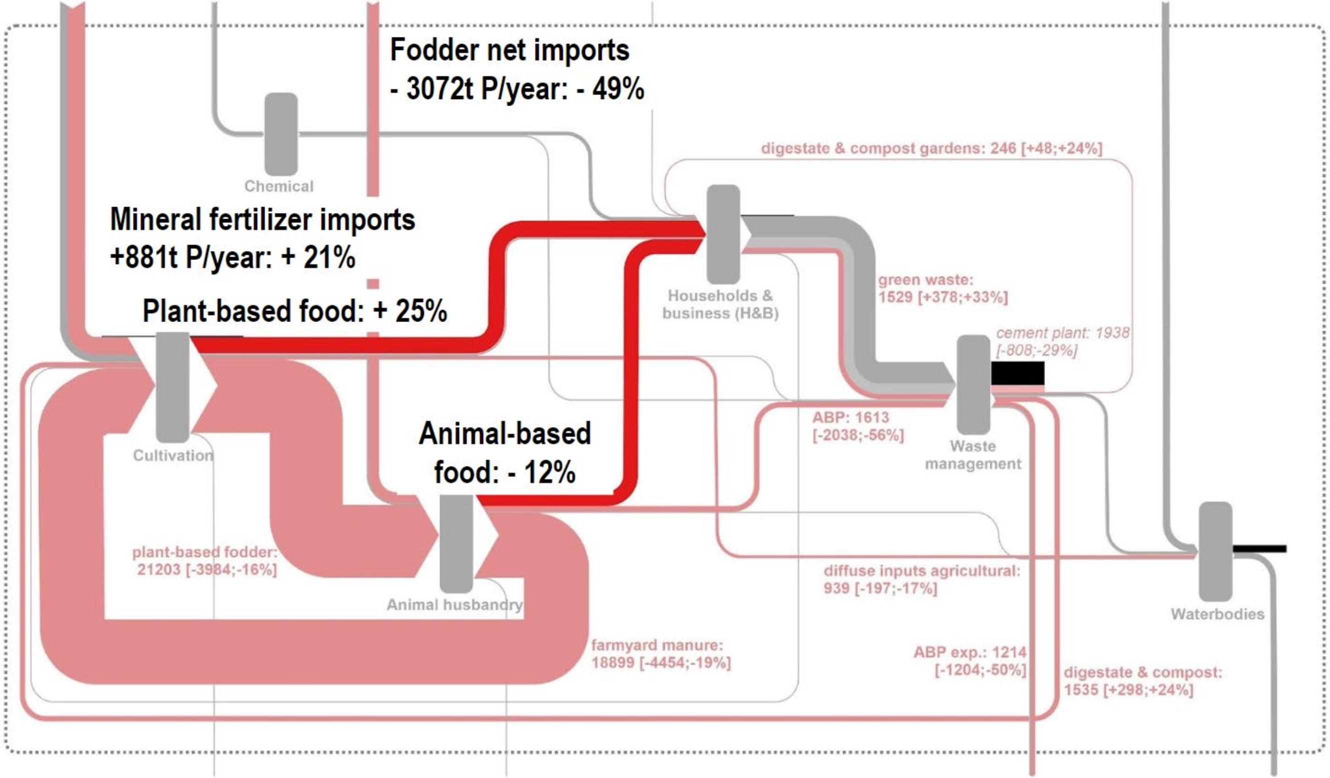
FSVO: Federal Food Safety and Veterinary Office
VVEA: Ordinance on Avoidance and Disposal of Waste

Balanced and healthy diets (landscape)

Recommended food consumption compared to consumption in 2014/15 according to FSVO

	%
Meat and fish	-68.5
Eggs	-19.5
Milk and dairy products	+50.0
Grain/rice/potatoes	+25.0
Vegetables	+76.5
Fruits	+5.3
Vegetable oils/fats	-34.0
Animal fats	-74.4
Nuts/seeds	+100.0
Sugar	-75.0

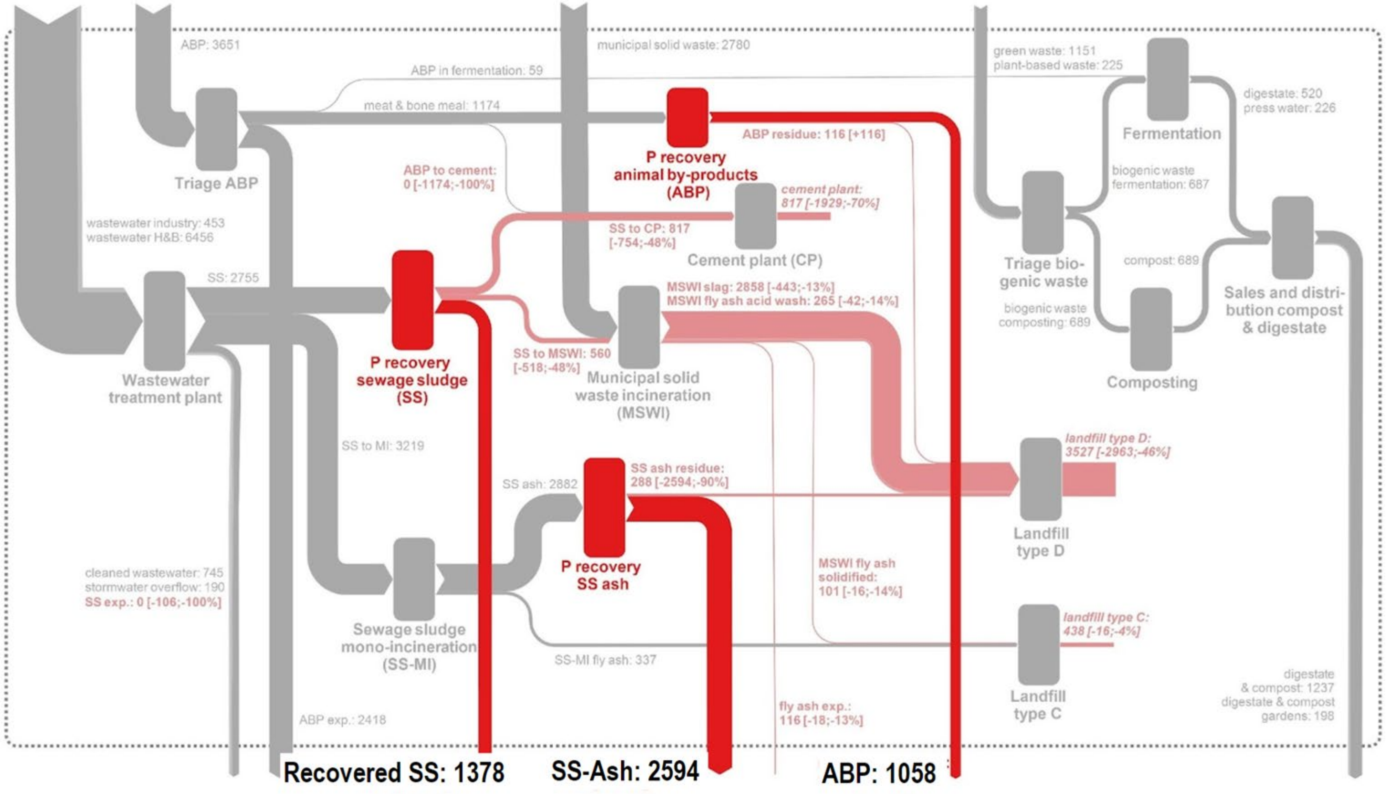
Scenario I: Balanced and healthy diets



→ Increase in dependency of imports of mineral fertilizers

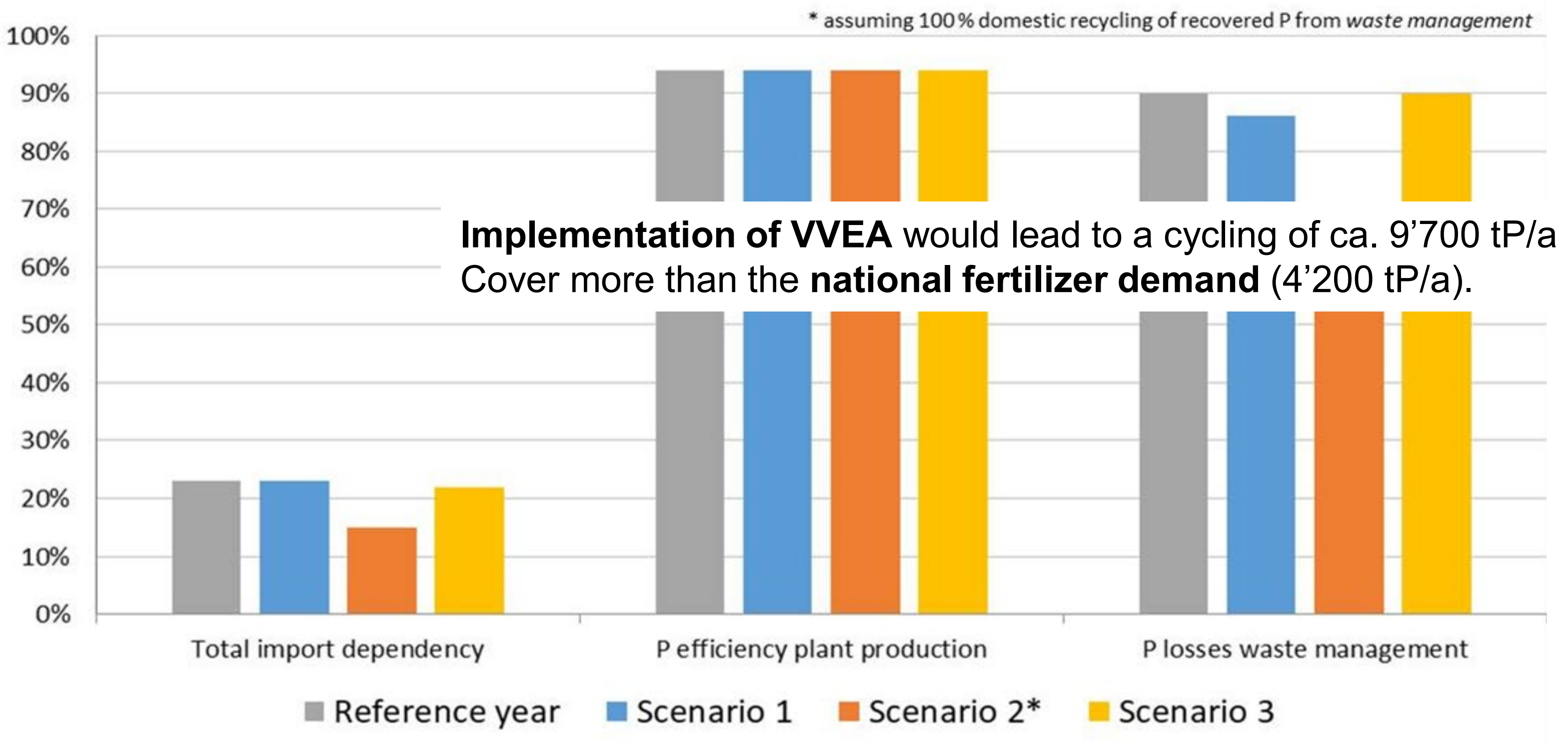
Source : Jedelhauser et al., 2018

Scenario II: VVEA implementation



Source : Jedelhauser et al., 2018

Institutional intervention triggers technological innovation



Implementation of VVEA would lead to a cycling of ca. 9'700 tP/a. Cover more than the **national fertilizer demand** (4'200 tP/a).

Scenario I: Balanced and healthy diets

Scenario II: VVEA implementation

Scenario III: Urine separation

Source : Jedelhauser et al., 2018

Using MFA to enable a circular economy

Minerals in the energy transition

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Swiss Energy Strategy 2050

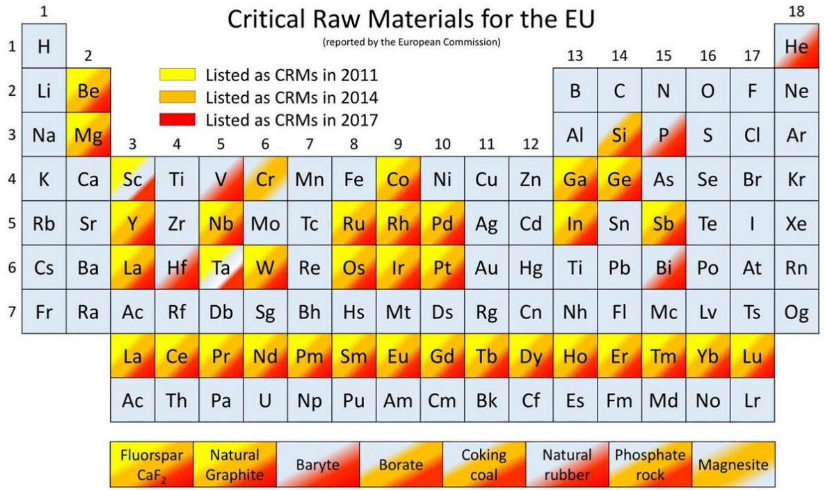
- Increase energy efficiency in buildings, mobility, industry
- Develop renewable energies (promotion, ease the legal framework)
- Phase out nuclear energy (no more construction of nuclear powerplants, security reasons)

To achieve targets:

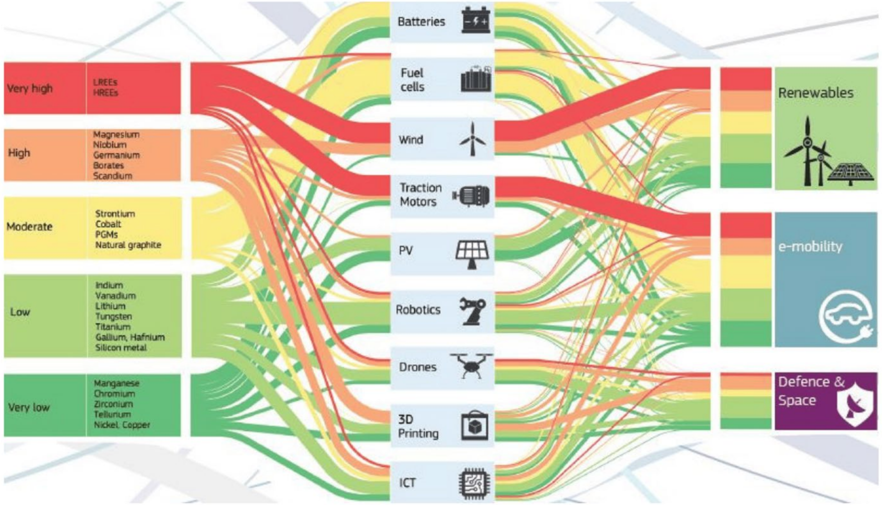
- Replace 70% of actual energy consumption by renewables
- Increase national energy production

Copper mine in South Africa (source: Dillon Marsh, "For what it's worth")

Critical raw materials



Rizzo, A.; Goel, S.; Grilli, M.L.; Iglesias, R.; Jaworska, L.; Lapkovskis, V.; Novak, P.; Postolnyi, B.O.; Valerini, D. The Critical Raw Materials in Cutting Tools for Machining Applications: A Review. *Materials* **2020**, *13*, 1377. https://doi.org/10.3390/ma13061377

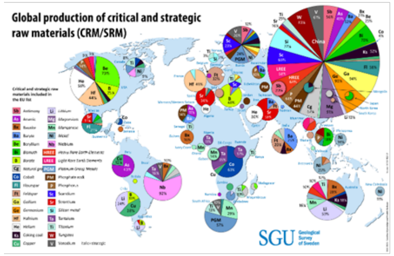


Bobba, S., Carrara, S., Huisman, J., Mathieux, F. et al., *Critical raw materials for strategic technologies and sectors in the EU – A foresight study*, Publications Office, 2020, https://data.europa.eu/doi/10.2873/58081

Approach: Dynamic MFA

Focus on solar panels, wind turbines and electric vehicles

2023 CRM material intensities ✖ Current installed capacities



Current CRM stocks in the Swiss renewable energies system

2050 CRM material intensities ✖ 2050 installed capacities

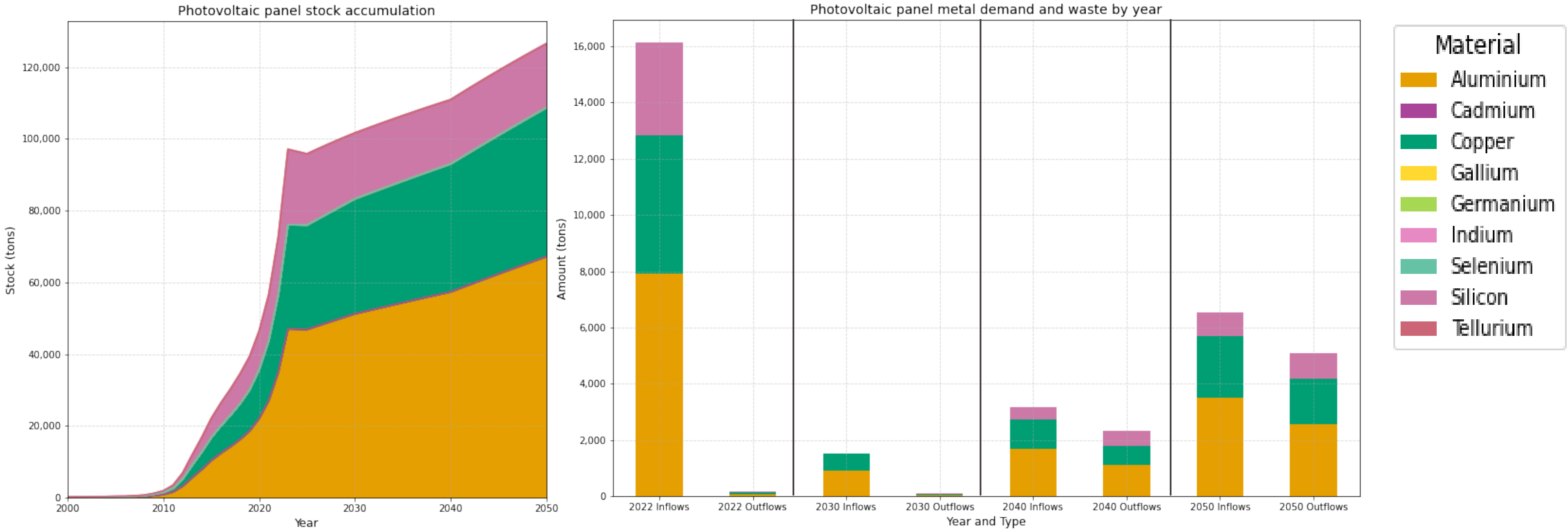
SURE scenarios

- SPS1: Team Sprint – Focus on Sustainability: World gradually implements green strategies; High regional and energy market integration; Social behavior and lifestyles supporting sustainability actions
- SPS2: Mountain Hike – Focus on Energy Security: World gravitates toward a multi-polar order; Regional conflicts increase energy security concerns; Social behaviour and lifestyles willing to "pay for more security"
- SPS3: Single Trail Run – Fragmented Regions: Regions implement climate policies at different speeds; Moderate regional and energy market integration; Social behaviour and lifestyles supporting local markets
- SPS4: Walk & Talk – Current Trends and Policies: World follows a path not markedly different from today; Geopolitical situation as of today; Social behaviour and lifestyles in favor of proven options and norms

2050 CRM needs for the energy transition according to different scenarios ✖ LCA impacts

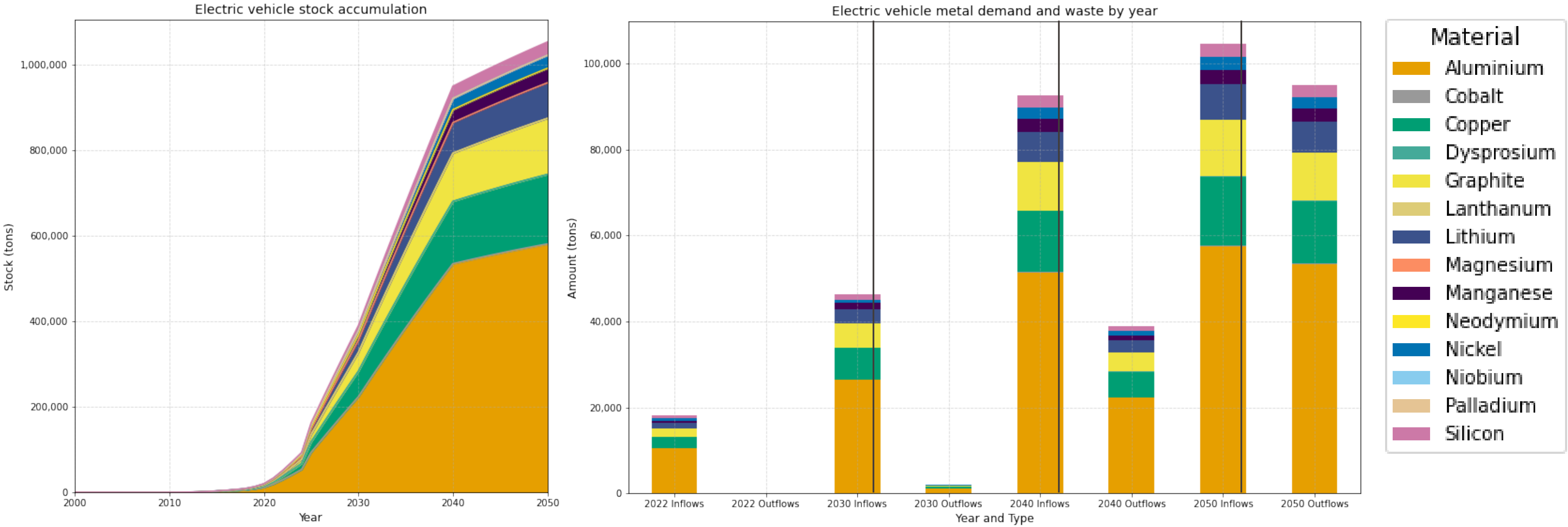
Léa Bitard (2024) and Corentin Mottet (2024)

Material stock and flows (PV)
2000 - 2050



Singhvi, Mottet, Bitard and Binder (2024)

Material stock and flows (EV)
2000 - 2050



Singhvi, Mottet, Bitard and Binder (2024)

Trade-offs between energy transition and material extraction

- A energy transition towards renewables is necessary for meeting decarbonisation targets
- However it comes with increased extraction and dependency on critical raw materials, with important environmental impacts
- As technologies reach the end of their lifespan in the next ten years, the question stands:
- To what extent can we close material loops?

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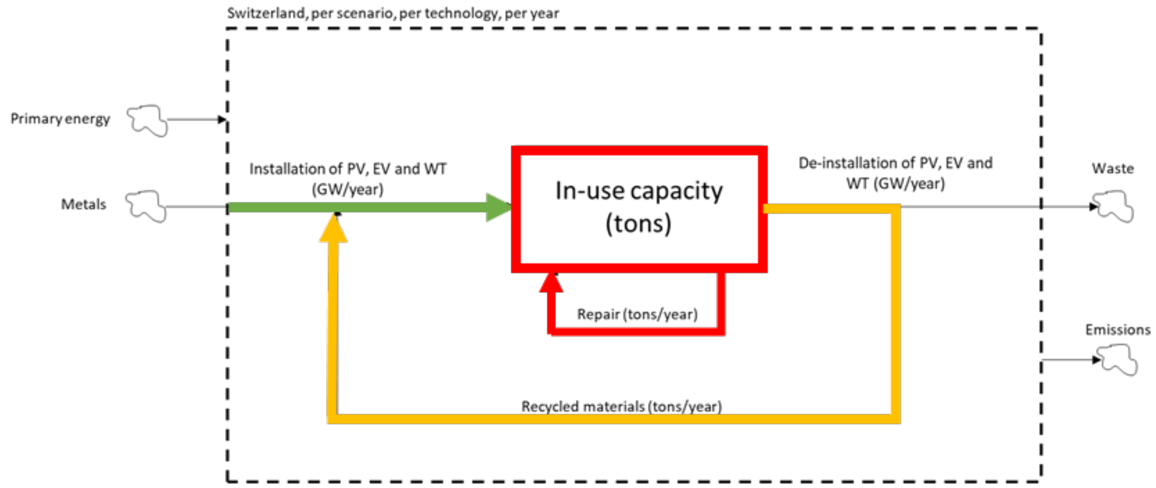
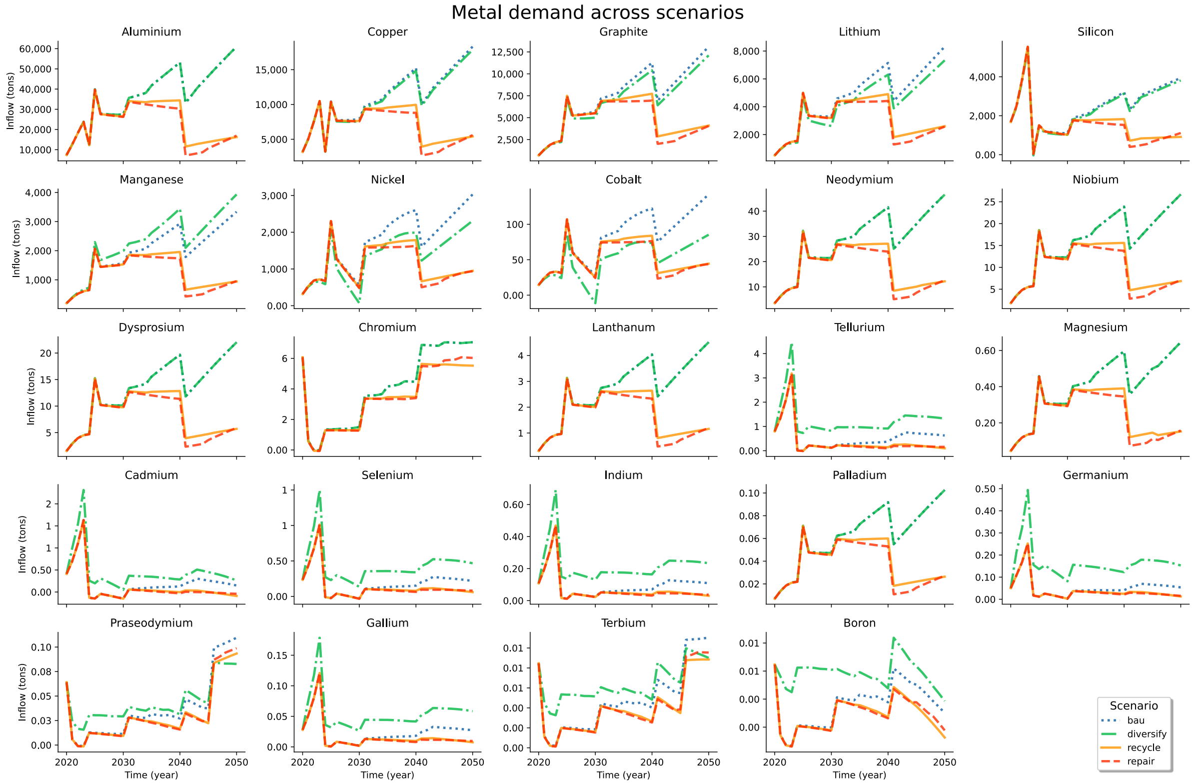
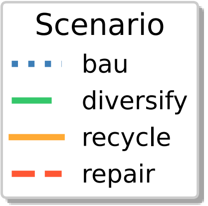
Circular economy scenarios



R0 Refuse	R1 Rethink	R2 Reduce	R3 Reuse	R4 Repair	R5 Refurbish	R6 Remanufacture	R7 Repurpose	R8 Recycle	R9 Recover

Circular economy scenarios





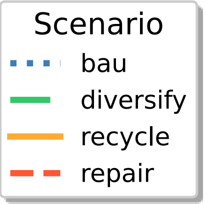
Singhvi, Mottet, Bitard and Binder (2024)

Circular economy scenarios

Business as usual: Pressure on rare earth metal demand for WTs shows annual increase

Local recycling capacity: Large components with long lifespans

Repair, reuse: Increasing lifespans by ten years reduces metal demand in the long term for most metals



Rethink / Reduce Diversification : Changing the share of sub-technologies shifts demand to alternative metals

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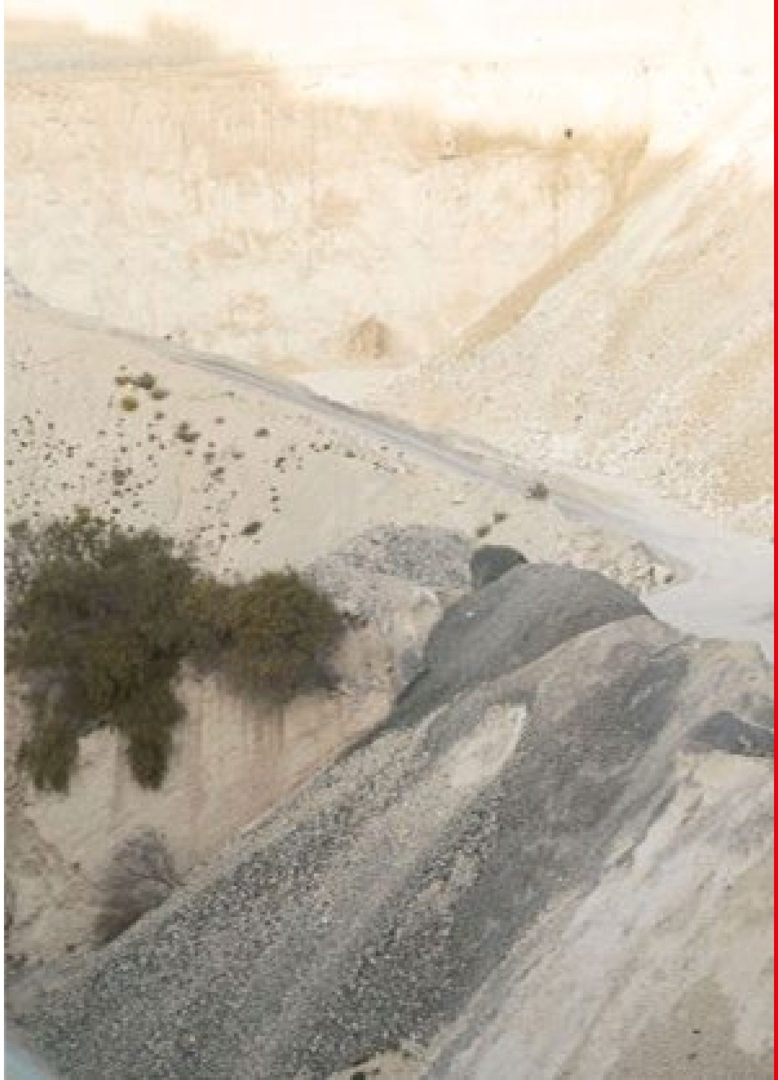

Summary

Laboratory on Human-Environment Relations in Urban Systems

Summary (I)

- Material flow analysis (MFA) is a systemic method based on mass balance
- It quantifies (the dynamics of) stock and flows over time
- It supports
  - Prioritization of measures and areas of innovation
  - Strategic decisions at national level
  - Circularity decisions in energy transition
  - Early detection of problematic developments

Summary (II)

- Reduction of environmental impact relies on changes in lifestyle, institutional innovation, and technical innovation
- It requires (several iterations of)
  - Scientific evidence
  - Public awareness
  - Policy
  - Technological innovation
- MFA can be a tool for:
  - Providing scientific evidence
  - Creating awareness
  - Monitoring

Thanks to:
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sweet swiss energy research for the energy transition

SURE

Schweizerische Eidgenossenschaft
Confédération suisse
Confederazione Svizzera
Confederaziun svizra

Swiss Federal Office of Energy SFOE

The future will be shaped by what we value, what we envision, and who surrounds us.

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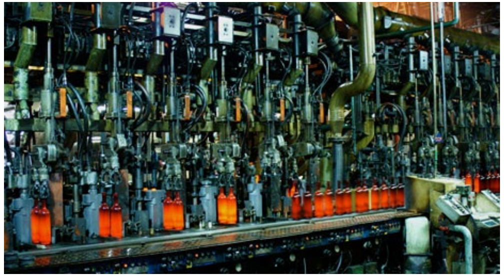
Additional method slides

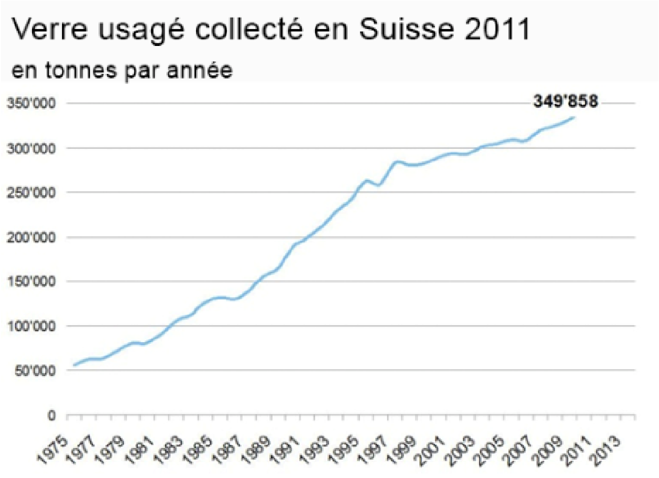
The case of glass recycling

Glass recycling: situation in Switzerland

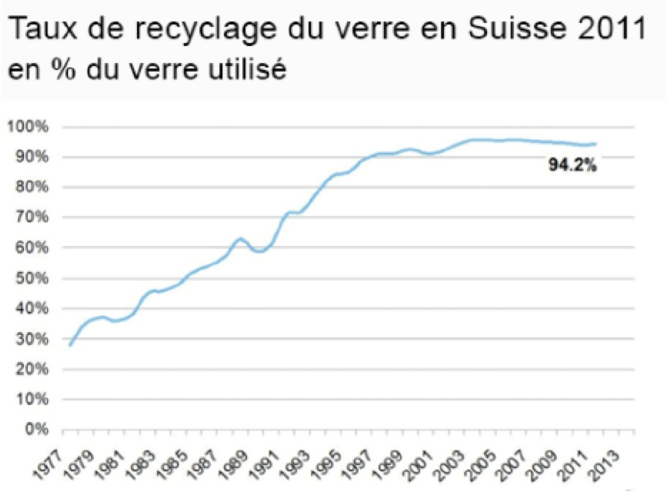


vetroswiss

... pour un recyclage efficace du verre ...

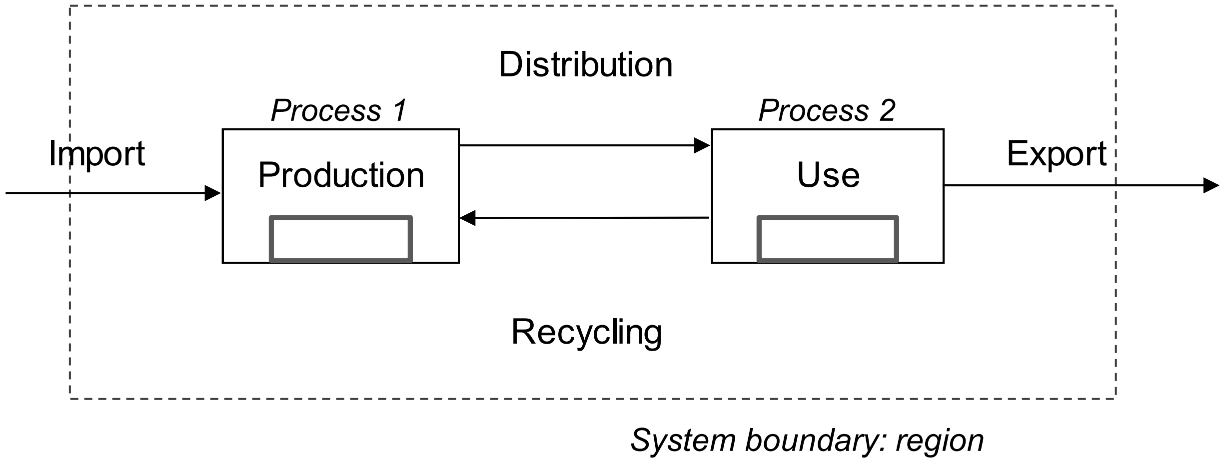


Evolution du verre usagé collecté dans toute la Suisse (Source : www.vetrorecycling.ch)



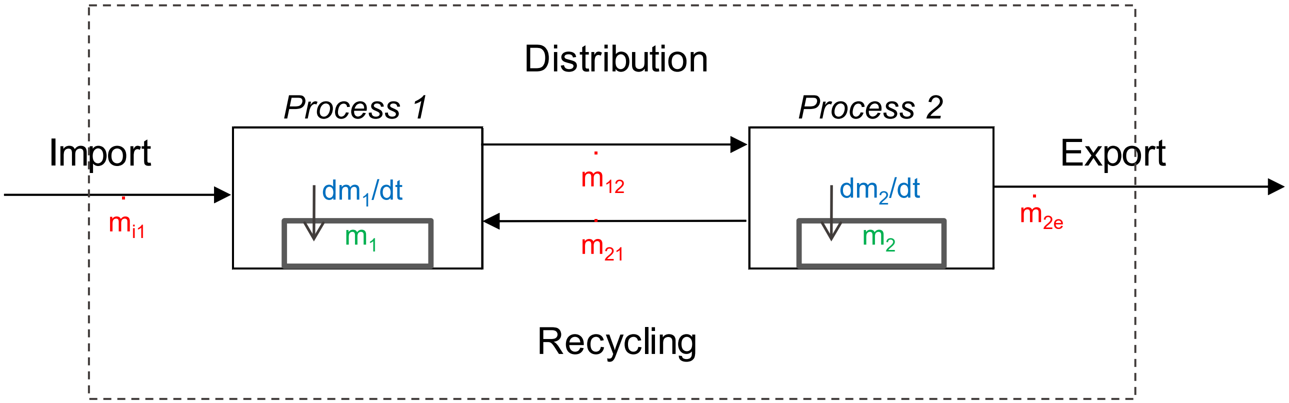
Evolution du taux de recyclage du verre en Suisse (Source : www.vetrorecycling.ch)

Glass recycling: the system



How many unknowns for the general case?
How many in steady state?

Glass recycling: solution



System boundaries: region

8 in general

6 in steady state

Glass recycling: solution

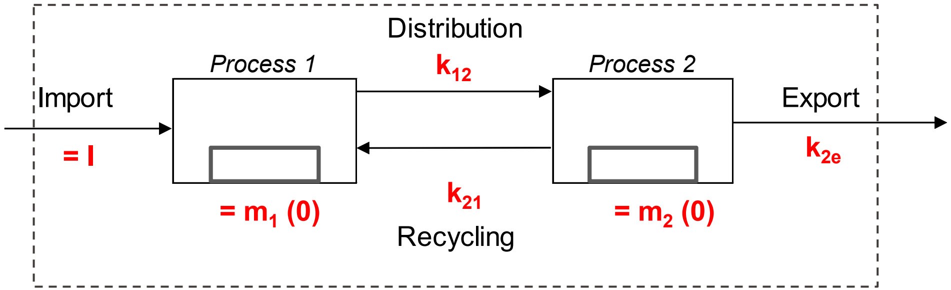
$$\frac{dm_1}{dt} = m_{i1} + m_{21} - m_{12}$$

$$\frac{dm_2}{dt} = m_{12} - m_{21} - m_{2e}$$

Another 6 equations needed for a finite number of solutions
=> System parameter equations

Glass recycling: solution

P1 = m_1 (0)	Initial value of the stock
P2 = m_2 (0)	Initial value of the stock
P3 = I	Input flow
P4 = k_{12}	Transfer coefficient
P5 = k_{21}	Transfer coefficient
P6 = k_{2e}	Transfer coefficient



Glass recycling: solution

Solution in steady state:

- $\mathbf{dm_1/dt = 0}$
- $\mathbf{dm_2/dt = 0}$
- $m_{i1} = I$
- $k_{12} = m_{12}/(m_{i1}+m_{21})$
- $k_{21} = m_{21}/m_{12}$
- $k_{2e} = m_{2e}/m_{12}$
- $\mathbf{m_1 = m_1(0)}$
- $\mathbf{m_2 = m_2(0)}$

Linear system of 4 equations and 4 unknowns

Glass recycling: solution

Solution in steady state:

- $\mathbf{dm_1/dt = 0}$
- $\mathbf{dm_2/dt = 0}$
- $m_{i1} = I$
- $k_{12} = m_{12}/(m_{i1}+m_{21})$
- $k_{21} = m_{21}/m_{12}$
- $k_{2e} = m_{2e}/m_{12}$
- $\mathbf{m_1 = m_1(0)}$
- $\mathbf{m_2 = m_2(0)}$

$m_{i1} = I$

$m_{12} = k_{12}I/(1-k_{12}k_{21})$

$m_{21} = k_{21}k_{12}I/(1-k_{12}k_{21})$

$m_{2e} = k_{12}k_{2e}I/(1-k_{12}k_{21})$

Glass recycling: solution

Numerical example in steady state:

- $\mathbf{dm_1/dt = 0}$
- $\mathbf{dm_2/dt = 0}$
- $m_{i1} = 19$ [kg/cap.year]
- $k_{12} = 1$
- $k_{21} = 0.87$
- $k_{2e} = 0.13$
- $\mathbf{m_1 = m_1(0)}$
- $\mathbf{m_2 = m_2(0)}$

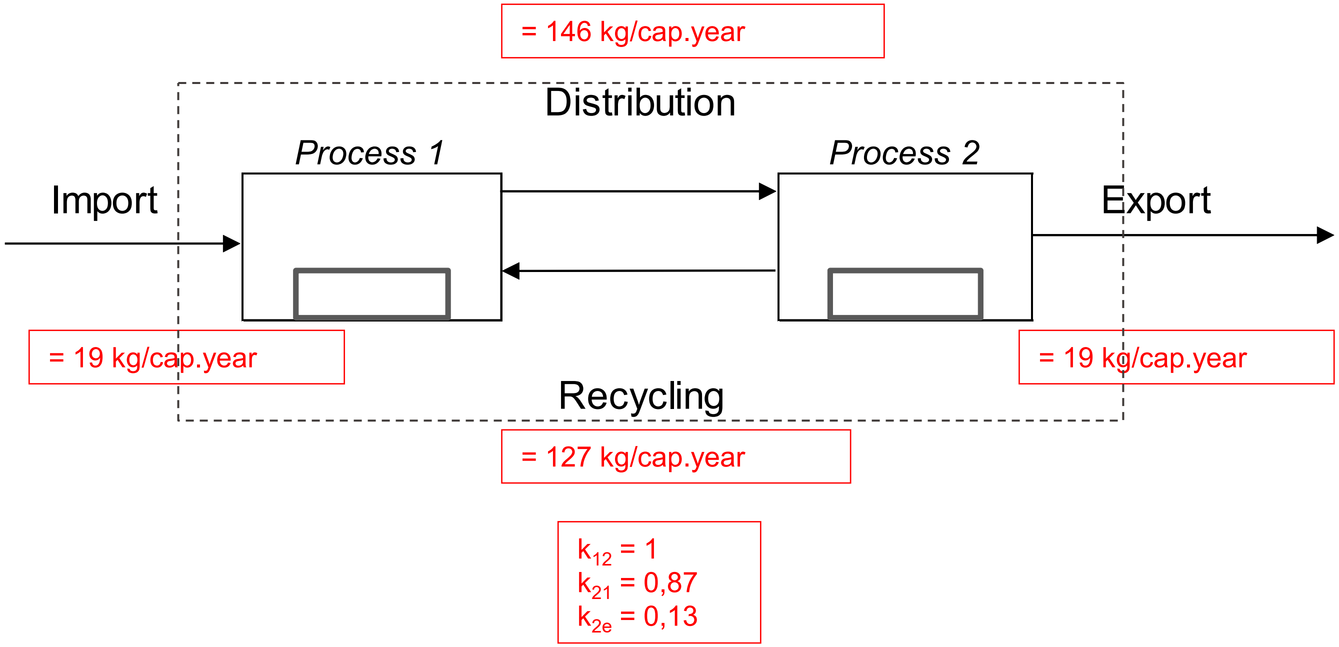
$m_{i1} = I$

$m_{12} = k_{12}I/(1-k_{12}k_{21})$

$m_{21} = k_{21}k_{12}I/(1-k_{12}k_{21})$

$m_{2e} = k_{12}k_{2e}I/(1-k_{12}k_{21})$

Glass recycling: solution



Glass recycling: validation

How to check your results?

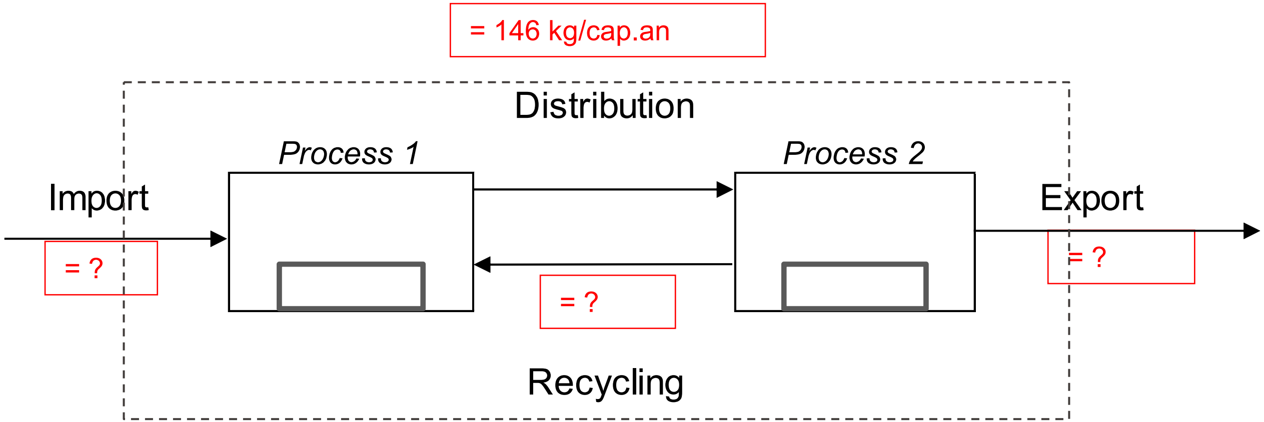
1) Compare with statistical data and time series

2) Compare with literature references

3) Get feedback from an expert in the field

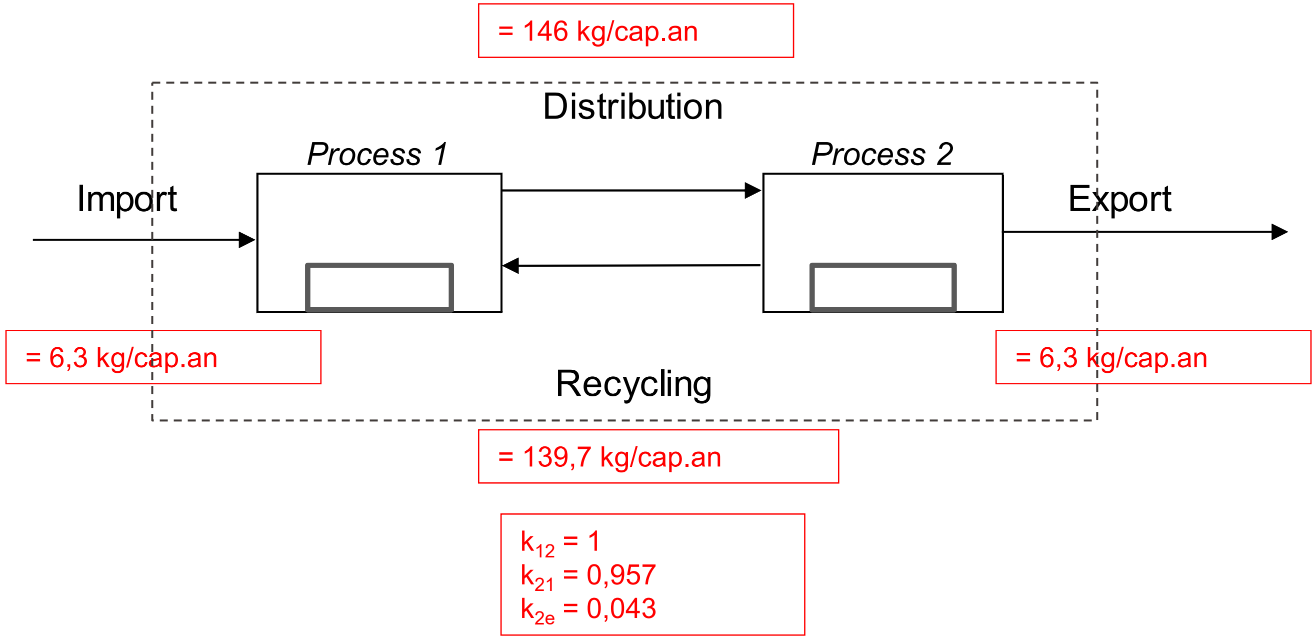
Glass recycling: validation

Glass recycling: what if k_{21} increases by 10%

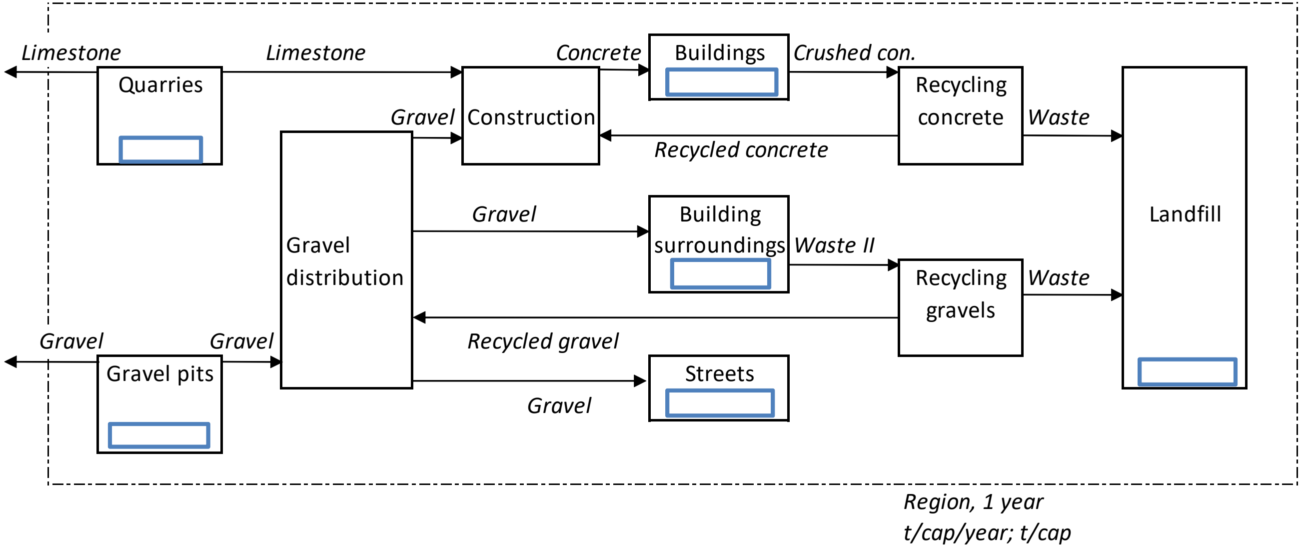


Glass recycling: validation

Glass recycling: what if k_{21} increases by 10%

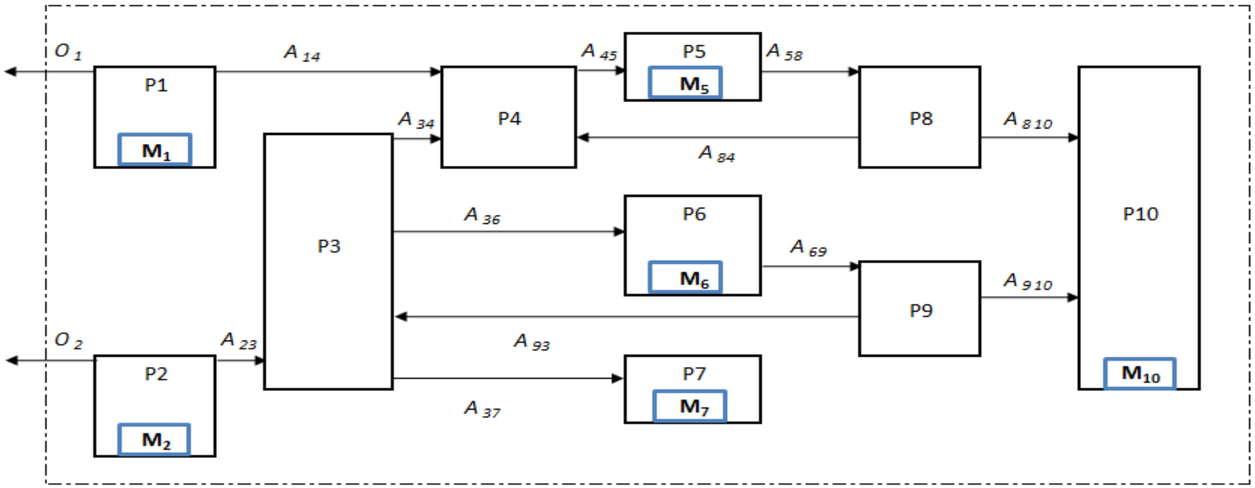


More complicated example: gravel



Michael Redle, Kies- und Energiehaushalt urbaner Regionen in Abhängigkeit der Siedlungsentwicklung, DISS. ETH Nr. 13108.

Example: model definition



Region, 1 year
t/cap/year; t/cap

Solution: matrix form

26 variables

10 equations

16 parameters (p_1 to p_{16}):

- Exports are known: 2 equations, p_1 and p_2
- Stocks are known: 6 equations, p_3 to p_8
- Transfer coefficients are known for construction (p_9), recycling concrete (p_{10}), recycling gravel (p_{11}).
- Etc...
  - => MFA in matrix form

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Solving MFAs in matrix form (1)

Mathematical – physical description of a system

Matrix form – input output analysis

=> Choice of the method depends on the type of system and the type of problem.

Solving MFAs in matrix form (2)

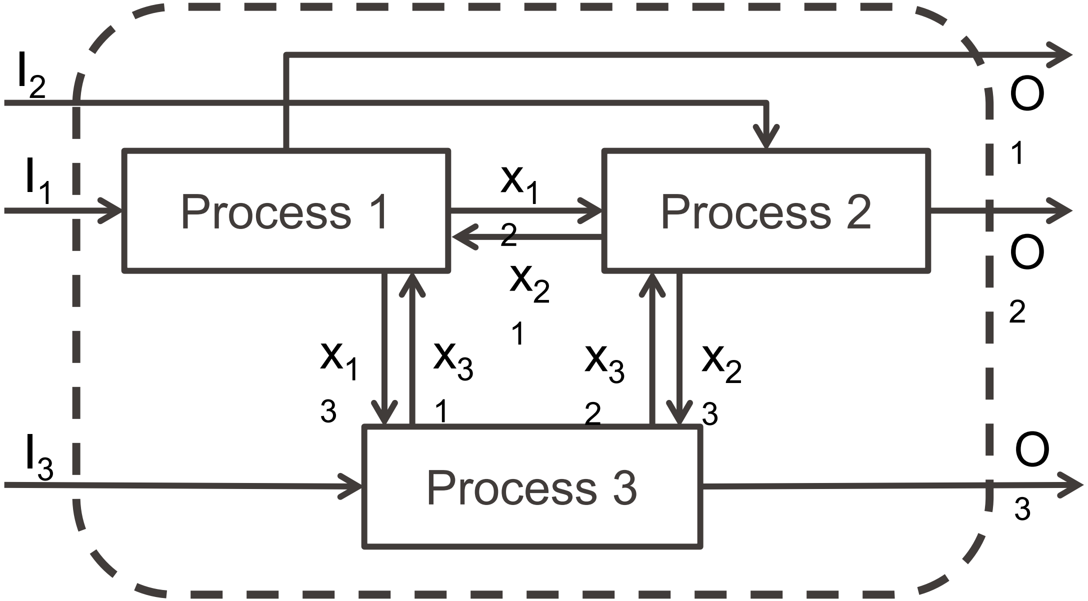
Key question:

What is the impact of a change in the inputs on the outputs ?

Example:

Imagine the cement manufacturing process. What effects has the substitution of primary fuels with alternative fuels on the quality of cement ?

Solving MFAs in matrix form (3)



Simplified system and notation!

Reading matrix formulation

	Process 1	Process 2	Process 3	Output	Total
Process 1	x_{11}	x_{12}	x_{13}	O_1	X'_1
Process 2	x_{21}	x_{22}	x_{23}	O_2	X'_2
Process 3	x_{31}	x_{32}	x_{33}	O_3	X'_3
Input	I_1	I_2	I_3		
Total	X_1	X_2	X_3		

Reading matrix formulation

	Process 1	Process 2	Process 3	Output	Total
Process 1	x_{11}	x_{12}	x_{13}	O_1	X'_1
Process 2	x_{21}	x_{22}	x_{23}	O_2	X'_2
Process 3	x_{31}	x_{32}	x_{33}	O_3	X'_3
Input	I_1	I_2	I_3		
Total	X_1	X_2	X_3		

= 0

= 0

= 0

x_{ij} = Flow from process i to process j

I_i = Input in process i

O_i = Output from process i

X_i = Sum of flows into process i

X'_i = Sum of flows out of process i

Solving MFAs in matrix form (4)

1. Set up the matrix and the balance equations for the flows
2. Set up the matrix and the balance equations for the transfer coefficients
3. Write the balance equations in matrix form
4. Solve the equation by matrix inversion

Basic assumption: steady state

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Matrix form and balance equations (1)

	Process 1	Process 2	Process 3	Output	Total
Process 1	= 0 x_{11}	x_{12} = 0	x_{13} = 0	O_1	X'_1
Process 2	x_{21}	x_{22}	x_{23}	O_2	X'_2
Process 3	x_{31}	x_{32}	x_{33}	O_3	X'_3
Input	I_1	I_2	I_3		
Total	X_1	X_2	X_3		

1. Which line equals which column in steady-state ?
2. Write the balance equation for process 1

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Matrix form and balance equations (2)

	Process 1	Process 2	Process 3	Output	Total
Process 1	x_{11}	x_{12}	x_{13}	O_1	X'_1
Process 2	x_{21}	x_{22}	x_{23}	O_2	X'_2
Process 3	x_{31}	x_{32}	x_{33}	O_3	X'_3
Input	I_1	I_2	I_3		
Total	X_1	X_2	X_3		

Steady state assumption: what goes in must come out

Balance equation:

$$x_{11} + x_{21} + x_{31} + I_1 = X_1 \quad = \quad x_{11} + x_{12} + x_{13} + O_1 = X'_1$$

Transfer coefficients equations

With:

$$x_{11} + x_{21} + x_{31} + I_1 = X_1$$

We know: $TC_{ij} = k_{ij} = x_{ij}/X_i$

We get:

$$X_1 * k_{11} + X_2 * k_{21} + X_3 * k_{31} + I_1 = X_1$$

Balance equations and matrix form (1)

$$X_1 * k_{11} + X_2 * k_{21} + X_3 * k_{31} + \ldots + X_n * k_{n1} + I_1 = X_1$$

$$X_1 * k_{12} + X_2 * k_{22} + X_3 * k_{32} + \ldots + X_n * k_{n2} + I_2 = X_2$$

…

$$X_1 * k_{1n} + X_2 * k_{2n} + X_n * k_{3n} + \ldots + X_n * k_{nn} + I_n = X_n$$

Matrix form:

$$K*X + I = X$$

Balance equations and matrix form (2)

$$K^*X + I = X$$

$$I = X - K^*X$$

$$I = A^*X - K^*X = (A - K)^*X$$

$$X = (A - K)^{-1} {}^*I$$

Note $A = \mathrm{diag}(1,1,...,1)$

Matrix of transfer coefficients

	Process 1	Process 2	Process 3	Output	Total
Process 1	k_{11}	k_{12}	k_{13}	k_{1O}	1
Process 2	k_{21}	k_{22}	k_{23}	k_{2O}	1
Process 3	k_{31}	k_{32}	k_{33}	k_{3O}	1

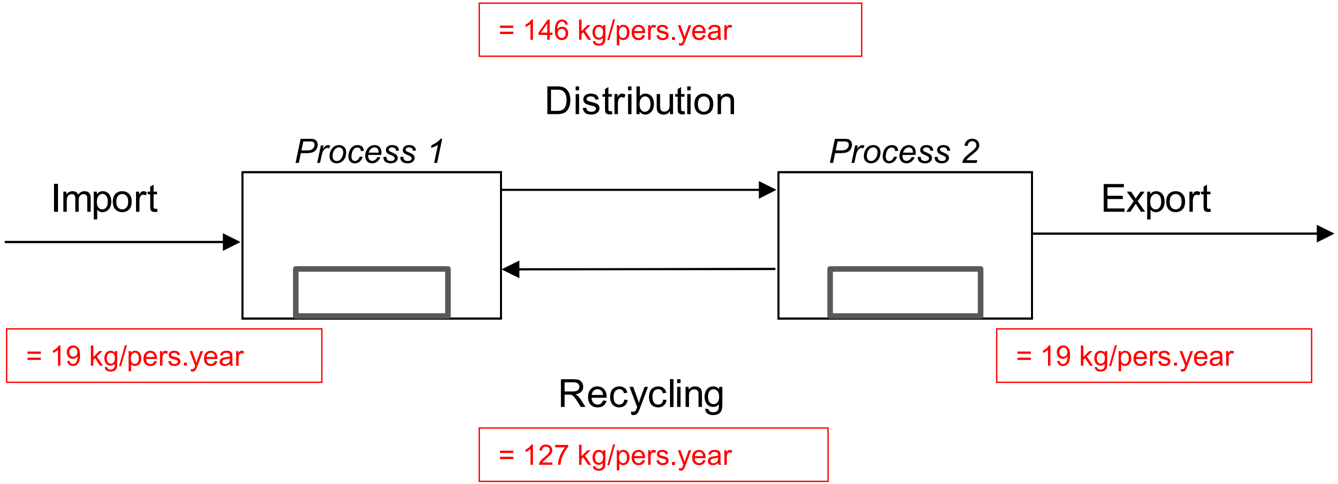
K^T

K = k_{11} k_{21} … k_{n1}

k_{12} k_{22} k_{n2}

… …

Glass example in matrix form



$k_{12} = 1$
$k_{21} = 0,87$
$k_{2e} = 0,13$

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Glass example in matrix form

	Production	Use	Export	Total
Production	0	146	0	146
Use	127	0	19	146
Import	19	0		
Total	146	146		

Matrix of transfer coefficients

	Production	Use	Export	Total
Production	0	1	0	1
Use	0.87	0	0.13	1
Import	0.13	0		
Total	1	1		